



# LIGHT AND MATTER

A NEW CLASSICAL THEORY OF LIGHT AND  
MATTER BASED ON THE MAXWELL EQUATIONS  
AND THE SPECIAL RELATIVITY THEORY WITH  
CRITICISMS OF THE EXISTING THEORIES

BY

B. M. SEN, M.A. (CAMB.), M.Sc., F.N.I.

*Smith's Prizeman at the University of Cambridge,  
Retired Professor, Calcutta University, Formerly  
Principal, Presidency College,  
Calcutta*



UNIVERSITY OF CALCUTTA

1958



*Agents in Europe*

Messrs. LUZAC & CO., LTD.,  
46, Great Russell Street, London, W.C., 1

535-1

se 474

BCU 2786

FIRST EDITION, 1948

SECOND EDITION (*Revised*), 1958

GS2109

PRINTED IN INDIA

PRINTED AND PUBLISHED BY SIBENDRANATH KANJILAL,  
SUPERINTENDENT, CALCUTTA UNIVERSITY PRESS,  
48, HAZRA ROAD, BALLYGUNGE, CALCUTTA.

1977B—15.5.1958—A.



## CONTENTS

	PAGE
CHAPTER I. THEORIES OF LIGHT—HISTORICAL AND CRITICAL	... 1
CHAPTER II. WHAT IS MATTER	... 19
CHAPTER III. A NEW THEORY OF LIGHT	... 36
CHAPTER IV. A NEW THEORY OF MATTER	... 53
CHAPTER V. SUMMARY AND CONCLUSIONS	... 68



## PREFACE TO THE FIRST EDITION

My paper, *A New Classical Theory of the Photon and the Electron* was published in Vol. XII, No. 6 of the PROCEEDINGS OF THE NATIONAL INSTITUTE OF SCIENCES OF INDIA in August, 1946; a short summary had been published in SCIENCE AND CULTURE of January, 1946. Though prints were widely distributed among the leading physicists, the learned societies and the scientific journals of India, Europe and America with an open invitation for comment and specific criticism, the response was disappointing. A few courteous acknowledgments were received but no criticism, constructive or destructive.

In the present booklet, the objections to the current theories have been explained in greater detail, supported by extensive quotations from well-known authorities, and the main thesis itself has been expounded more fully. It has been my endeavour to present the case against the existing theories and formulate the new theories in as readable a form as possible. The mathematics is fairly simple and will, it is hoped, be intelligible even to those physicists whose mathematical equipment may not be high.

It may be stated here that the thesis has no conflict with Quantum Mechanics regarded from a statistical point of view, though there is a certain amount of divergence. Even if the present theory be accepted, Quantum Mechanics will not have to be scrapped, but will have to be re-written from a new standpoint.



I shall feel grateful for any comment or criticism which may be offered. Any expression of approval or suggestion for improvement will be doubly welcome. In any case, it behoves the Quantum physicists to meet the criticisms levelled against them. Truth is not served by ignoring criticism. I trust, however, that argument of the type " Fifty thousand Frenchmen cannot be wrong " will not be brought forward, either explicitly or by implication.

12, BALLYGUNGE CIRCULAR ROAD,  
CALCUTTA, *18th June, 1947.*

B. M. SEN



## PREFACE TO THE SECOND EDITION

THE reception to the booklet has not been enthusiastic—not that there was any fault or defect found in the main arguments. Requests for comments and criticisms to physicists and scientific periodicals excited no response. But mere shaking of the head however wise is not criticism. Only one defect was pointed by the Editor of a well known journal, *viz.*, the assumption of the invariance of the electromagnetic vector under Lorentz transformation, while the invariance of the Maxwell equations was the proposition proved. But when the lacuna was filled in a subsequent paper, the journal refused its publication as unsuitable! The fact is that those who would be judges are themselves deeply committed to the current theory. Naturally they are loath to accept the challenge and enter into arguments. One sympathetic physicist offered the consolation that Mendel's work was recognised fifty years after his death!

The only gesture of encouragement which is gratefully acknowledged came from a mathematician, Prof. Birkhoff who as Chairman of the International Congress of Mathematicians at Harvard in 1950, recommended my name for a special invitation. But the invitations had gone out and there was no vacant place. I received also invitations from the International Congress of Mathematicians at Amsterdam in 1954, and from the Seminars for Theoretical Physics in Aachen, Heidelberg, Cambridge and the University College, London. I thank the authorities for the courtesy.



In the circumstances, all that I can do is to embody the results obtained and leave the matter to the judgment of posterity. My three objections to the Einstein principle  $W = h\nu$  which is the basis of modern physics are still unanswered. They are:

(i) a divisible beam with an indivisible quantum of energy is a contradiction in terms, (ii) the theory that light behaves as a particle or a wave just to suit experimental needs, introduces the supernatural in the domain of natural philosophy, (iii) the existence of the continuous spectrum implies infinite radiated energy. These are logical defects and until they are answered, modern Physics can only be regarded as empirical, all its successes notwithstanding. The supposed verification of any theory is not always a reliable indication of its truth as has been proved in the case of Dirac's prophecy of the positron and Yukawa's of the meson. The scope of modern experimental physics is so vast that it is a very complicated jigsaw puzzle to formulate a theory to fit all known facts.

In this edition the booklet has been revised and in places, the arguments amended. Once again I invite comments and criticisms.

12, BALLYGUNGE CIRCULAR ROAD,

CALAUTTA—19,

7th May, 1958

B. M. SEN



## CHAPTER I

### THEORIES OF LIGHT—HISTORICAL AND CRITICAL

1. The various theories of light—the elastic solid theory of Fresnel and Young, the electromagnetic theory of Maxwell, the quantum theory of Planck and Einstein and the wave theory of Schrödinger, Heisenberg and Dirac—have one characteristic in common. They all emphasise their successes and pass lightly over their difficulties. Like the proverbial curate's egg, they are all good in parts. To get a clear concept of their failures and difficulties it is necessary to recount them with special emphasis. This is a necessary preliminary before the formulation of the successful theory.

When a theory does not explain, or if it runs counter to, facts of experiment, the usual practice in the scientific world is, or at least ought to be, to hold the theory in suspense until such difficulties are cleared. When any theory contradicts a generally accepted principle, the only logical procedure is to give up either the theory or the principle. But in present day physics, all the theories are accepted, each reigning supreme in its own limited sphere though contradicting the others in fundamentals. All efforts at co-ordination have had no marked success. The correspondence principle, seeking to bridge the gulf between the classical theory and the quantum theory,



accepting the former when convenient and rejecting it in other cases, without reconciling the fundamental concepts, has added to the confusion of the entire position. Modern physics has very conveniently forgotten the fundamental principle of logic that no theory may be half right and half wrong. There are no tight compartments in the brain for different physical theories.

2. As is well known, the elastic solid theory is eminently successful in the region of Physical Optics, though the fundamental assumption about the nature of production of light waves, *viz.*, vibration of electrons in an elastic solid medium dragging the jelly-like material after themselves, has long been given up. The facts of Geometrical Optics, reflection and refraction, are explained by any wave theory and those of Physical Optics—polarisation, interference and diffraction—are all explained simply and apparently conclusively by the elastic solid theory. It put itself into a practically impregnable position for those times (the end of the eighteenth century) by discovering and explaining diffraction and thus overcoming Newton's objection that waves must be capable of bending round straight edges, which apparently light waves cannot do.

But the theory fails abjectly when the energy of such waves is considered. For interference, it is essential that the same single beam must be divided into two components which are made to recombine after one of them has been retarded by half a wave-length. It is a matter of common experience that two separate beams cannot be made to interfere. But the energy of a beam, according to the



elastic solid theory, is proportional to the square of the amplitude of vibration. The amplitude of the original beam, on the other hand, is equal to the sum of the amplitudes of the component beams.\*

This means a violent clash with the principle of Conservation of Energy, with no means of escape. Unless physicists were prepared to give up the Principle of Energy, the only logical course would have been to regard the theory at least with some reservation. But curiously enough, this big difficulty is simply passed over and as far as our knowledge goes, no mention is ever made of it in any of the current text-books on Physical Optics. It is inconceivable that such an elementary and fundamental difficulty could have escaped the notice of physicists over so many years.

3. Maxwell's electromagnetic theory records a great achievement in establishing connection between several branches of Physics, light, electricity and magnetism hitherto regarded as absolutely independent of one another. Its main contribution is to specify the nature of the light waves while accepting in a vague way the explanations of

---

\* A physicist friend has pointed out that the amplitude of the original beam is equal to the vector sum of the amplitudes of the component beams and that a difference of phase is introduced by reflection. We have considered here the amplitude at the instant of separation of the two beams before the reflection actually takes place. Also vector addition does not bar out scalar addition which is a special case. In any case, it is not worth while to examine the point more thoroughly, as the theory itself has been given up, except for college teaching.

the facts of Physical Optics offered by the elastic solid theory. But it leaves the problem of energy in precisely the same position as before. If a beam with electric force  $E$  is split up into two beams with components,  $E_1$  and  $E_2$  in a free medium

$$E = E_1 + E_2.$$

The density of energy is proportional to  $E^2$ . In order that the principle of energy may hold in every case, we must have

$$E^2 = E_1^2 + E_2^2.$$

The two relations are obviously hopelessly contradictory. Curiously enough, this difficulty also finds no mention in any exposition of the Maxwell theory.

Jeans has given full mathematical working of refraction and reflection of a wave polarised in and perpendicular to, the plane of incidence (Jeans, II, p. 529 *et seq.*). He has shown that if the wave front of the incident wave is  $K_1(x \cos \theta_1 + y \sin \theta_1 - V_1 t)$ , *i.e.*, the electric and magnetic vectors involve the exponential factor  $\exp. iK_1(x \cos \theta_1 + y \sin \theta_1 - V_1 t)$ , the refracted wave  $\exp. iK_2(x \cos \theta_2 + y \sin \theta_2 - V_2 t)$  and the reflected wave  $\exp. iK_3(x \cos \theta_3 + y \sin \theta_3 - V_1 t)$ , then the condition of continuity at the boundary  $x=0$  gives

$$K_1 \sin \theta_1 = K_2 \sin \theta_2 = K_3 \sin \theta_3.$$

$$K_1 V_1 = K_2 V_2 = K_3 V_1.$$

The laws of refraction and reflection follow immediately. These laws can, however, be deduced from practically any undulatory theory of light.



There is no clear indication how the energy of the incident beam as well as of the refracted and reflected beams is to be calculated (*i.e.*, over the entire space or not). In any case it is certain that the energy condition is not satisfied. The energy of the incident beam cannot be proved to be equal to the sum of the energies of the refracted and the reflected beams in spite of Jeans' evident anxiety that the equality should hold (II, p. 531).

One reason can be put forward at once. If the Maxwell equations are correct for a dispersive medium, "there ought to be a single definite refractive index for each medium, whereas the phenomenon of dispersion shows that the refractive index of any medium varies with the wavelength of light. It is easy to trace this difficulty to its source. The phenomenon of dispersion is supposed to arise from the periodic motion of charged electrons associated with the molecules of the medium, whereas the theoretical value which has been obtained for the velocity of light has been deduced on the supposition that the medium is uncharged at every point" (Jeans, II, p. 525). In fact, the Maxwell theory for a dispersive medium is very obscure.

4. Towards the close of the nineteenth century, many experimental facts about the energy were discovered and the result was Planck's law, followed by the bold generalisation of Einstein that every beam of frequency  $\nu$  possesses a quantum of energy  $h\nu$ . The difference in the intensity of light is accounted for by the difference in the number of light quanta in the incident beam.

Sommerfeld quotes the following passage from Hertz' address at Heidelberg in 1889:



“ Was ist das Licht? Seit den Zeiten Youngs und Fresnels wissen wir, dass es eine Wellenbewegung ist. Wir kennen die Geschwindigkeit der Wellen, wir kennen ihre Länge, wir wissen dass es Transversalwellen sind; wir kennen mit einem Worte die geometrischen Verhältnisse der Bewegung vollkommen. An diesen Dinge ist ein Zweifel nicht mehr möglich, eine Widerlegung dieser Anschauungen ist für den Physiker undenkbar. Die Wellentheorie des Lichtes ist, menschlich gesprochen, Gewissheit.”

Sommerfeld proceeds “ Ist diese Gewissheit inzwischen erschüttert worden? Ja und Nein! In allen Fragen der Interferenz und Beugung hat die Wellenlehre nicht nur ihre Stellung behauptet, sondern sie hat neuen Boden gewonnen; sie hat sich ausgedehnt, nach der Seite der kleinen Wellenlängen bis hinab zu den Röntgen und  $\gamma$ -strahlen, nach der Seite den grossen Wellenlängen bis zu den kilometerwellen der drahtlosen Telegraphie. In allen Fragen aber, welche, um mit Einstein zu reden, die Erzeugung und Verwandlung des Lichtes betreffen, sind wir von der Unzulänglichkeit der Wellenoptik überzeugt.

“ Die lichtelektrischen Erscheinungen und die sonstigen Absorptionsvorgänge im optischen und Röntgengebiet erwecken durchaus den Eindruck, als ob die Wellenenergie ebenso wie die Energie korpuskularer Strahlung an einzelnen Stellen punktförmig konzentriert sei. Man kommt so zu dem extremen Standpunkt der ‘ Lichtquanten ’ die von dem Emissionszentrum aus mit Lichtgeschwindigkeit forteilen. . . . . Denselben Umsatz von Strahlungsenergie des Betrages  $h\nu$  in die Hubarbeit eines Elektrons sehen wir sich im Atomverbände beim optischen Absorptionsvorgänge vollziehen ” (Sommerfeld, pp. 50-51).



5. But the Einstein relation revives at once in an acute form the problem of interference which requires that the same beam must be split up into two components. From a perusal of most treatises on Quantum Mechanics one would not suspect that such a problem existed at all. It seems to have been systematically ignored. Dirac, however, offers the following explanation:—

“ Suppose we have a beam of light which is passed through some kind of interferometer, so that it gets split up into two components and the two components are subsequently made to interfere. We may . . . . . take an incident beam consisting of only a single photon and inquire what will happen to it as it goes through the apparatus. This will present to us the difficulty of the conflict between the wave and corpuscular theories of light in an acute form. . . . . We must now describe the photon as going partly into each of the two components into which the incident beam is split. . . . . For a photon to be in a definite state of motion it need not be associated with one single beam of light, but may be associated with two or more beams of light which are the components into which one original beam has been split ” (Dirac, p. 7).

Such an argument coming from a man of Professor Dirac's eminence is entitled to serious consideration, but it breaks down at the most superficial criticism. What happens when the two component beams are widely separated, say by millions of miles? The quantum of energy cannot be present in both as it is inconceivable that the seat of energy can be millions of miles away from the path of the beam. We must then have a beam of light



without energy! Obviously, no amount of sophistry can explain the divisibility of a beam as required by the phenomenon of interference and retain the indivisible quantum of energy as enunciated by Einstein.

6. The consequence has been a duality which pervades the entire structure of modern Physics. The so-called wave and particle characters of light are treated entirely separately and there is no bridge over the gulf. The present view seems to be that light is both particle and wave. Its particular manifestation depends on the setting of the apparatus used for the experiment. If we set up an interferometer, light behaves obligingly as waves of certain length, but if we use a photo-electric apparatus, light behaves as particles possessing certain mass and momentum. But the curious feature is that the two characters cannot be made the subject-matter of the same experiment. This is in strange contrast with the behaviour of nature in experiments designed to investigate the relative motion of ether and matter. Then "the forces of nature seemed without exception to be parties to a perfectly organised conspiracy to conceal the Earth's motion through the ether. This, of course, is the language of the layman, not the man of science. The latter prefers to say that the laws of nature make it impossible to detect the Earth's motion through the ether. The philosophical contents of the two statements are precisely identical" (Jeans, I, p. 88).

The current theory about the dual nature of light is only the obverse side of Jeans' picture. Here Dame Nature behaves as a kind obliging lady and not like a perverse and



wilful young woman. She, however, refuses to be hustled into doing two things at the same time! And this is the language not of the ignorant man in the street, but of eminent men of science. **Apparently physicists find nothing incongruous in the introduction of the supernatural in the domain of natural philosophy.**

Does this not suggest that the wave and particle properties are not combined in the same party, but belong to two different physical entities, which, however, can easily get mixed up and are difficult to separate? In any case, it is clear that the principle of duality is only a confession of ignorance without any trace of apology or humility befitting our shortcoming.

The present tendency is to regard the entire energy of a beam as concentrated in the photon, the electromagnetic wave acting merely as the guide. If the implication is that the beam is divisible while the photon is not, the existence of a beam without energy is admitted. This runs counter to Einstein's fundamental relation  $W = h\nu$ . Nor is it conceivable that an electromagnetic wave possesses no energy at all. This continually shifting ground is a sure indication of weak logical foundation.

7. It is noteworthy that modern Physics deals entirely with energy and momentum and has very little to do with the wave length. One important exception is the experiment on the diffraction of electrons by Thomson and others. It has accepted unreservedly Einstein's relation  $W = h\nu$  associating a quantum of energy  $h\nu$  with every wave of frequency  $\nu$  and placed it in an apparently impregnable position as one of the pillars on which rests the



structure of Quantum Mechanics. But it is yet to be explained why radio waves of length 15 to 400 metres can penetrate a solid wall while a beam of visible light of wave length of order  $10^{-5}$  cm possessing many hundreds of million times more energy according to the Einstein law cannot penetrate one millimetre of pasteboard!

8. Another objection to the Einstein law may be mentioned here. The finite breadth of the spectral lines and the existence of the continuous spectrum imply an infinite number of frequencies. To put it crudely, it is an elementary conclusion of the Theory of Sets of Points that a continuous line of finite length—it is immaterial whether the line is a millimetre or a kilometre in length—possesses an infinite number of points. They form what is technically called a non-enumerable set. Consequently, if every frequency carries a finite quantum of energy, the total energy radiated by a finite mass of excited radiating matter must be infinite. Not even the store of energy of the sun is sufficient to cover the expenditure of radiated energy through one Ångström unit of the continuous spectrum in one second! **It seems that in their preoccupation with complex experiments, physicists have lost sight of the dispersion of white light by a prism—an experiment familiar to every schoolboy.**

It must be emphasised that it is not possible to explain a continuous spectrum by imagining a large but finite number of discrete lines without sacrificing the Schrödinger equation, which is the foundation of modern wave mechanics. It admits of discrete as well as continuous eigenvalues implying the existence of discrete lines as well



## THEORIES OF LIGHT—HISTORICAL AND CRITICAL II

as of the continuous spectrum. It may be contended that the continuous eigenvalues solution of the Schrödinger equation gives only the probable lines of the spectrum, but does not imply that the full complement of radiations corresponding to every point in the continuous spectrum, is simultaneously present. In other words, the radiations, though apparently continuous, are really discontinuous and discrete as regards frequency and intermittent as regards time. Even then the position cannot be saved. Suppose we have one gram of excited matter, radiating energy in the form of a continuous spectrum—the whole operation taking a finite length of time, say five minutes. If during this interval, every radiation in the range has been emitted, it is clear that the total radiated energy is infinite. Nor is there any jot of evidence to show that a continuous spectrum really possesses discontinuity in frequency.

9. The current Quantum Mechanical explanation of the finite breadth of the spectral lines leaves a good deal of room for objections on logical grounds. The classical explanation (Heitler, p. 113) proceeds on the assumption of radiation by a vibrating electron. It is well known that such assumption has long been discarded. In fact it is one of the corner-stones of Quantum Mechanics<sup>1</sup> that an electron radiates energy only when it changes from one energy level to another. If a vibrating electron could emit radiation, the Rutherford Bohr model of an atom would be an impossibility, for an orbital motion can be resolved into two simple harmonic oscillations.

Then the intensity distribution in a broad spectral line is found from probability considerations and the formula



obtained identified with that obtained from classical considerations (Heitler, p. 113). But finding the law of variation of intensity is a problem absolutely different from that of finding an explanation for the finite breadth. Finding the law of motion of a projectile is not the same as explaining the motion and gravitation. Also, the application of the correspondence principle when the underlying physical ideas are absolutely contradictory, seems to be extremely illogical. By way of comparison and contrast, it may be pointed out that the relation between Einstein's theory of planetary motion and the Newtonian theory is of a different character—the latter being only an approximation of the former. Einstein's idea of a field of force though distinct from that of Newton does not contradict it.

10. The logical explanation of the breadth of the spectral lines is based on Heisenberg's uncertainty principle (Heitler, p. 113)

$$\Delta E \Delta t = h$$

which states that the uncertainty in the determination of energy and time (whether of emission or reception is not clear, probably the latter, since observations and not concepts are the concern of Quantum Mechanics) are complementary. But in investigating the theory of the breadth, we are not concerned with the accurate determination of the time at all. So there ought not to be any limit to the accuracy of observation of the frequency. Further, this explanation ignores the variation in the breadth of the different lines. It offers no semblance of explanation of



the fact that the D-line of Sodium, for example, is so much broader than many other lines of the spectrum. If there be any substance in all these objections, **Einstein's relation  $W = h\nu$  must be regarded as an over-generalisation.**

11. Again Dirac defines his  $\delta$ -function as follows:—

$$\delta(x) = 0 \text{ when } x \neq 0, \quad \int_{-\infty}^{\infty} \delta(x) dx = 1. \quad (\text{Dirac, p. 72})$$

This function plays an important role in Quantum Mechanics.

Taken literally the integral does not make sense, for an integral must cover a stretch of the independent variable which is more than a mere point. If we extend the definition by making  $\delta(x) = 0$  when  $x$  does not lie between  $\pm \epsilon$  and then make  $\epsilon \rightarrow 0$ , it is easily verified that  $\delta(x)$  cannot exist. Let us suppose that the indefinite integral of  $\delta(x)$  is  $f(x)$ . Then

$$f(+\epsilon) - f(-\epsilon) = 1.$$

If we make  $\epsilon \rightarrow 0$ , we get  $f(+0) - f(-0) = 1$ , which makes  $f(x)$  discontinuous at the origin. It cannot, therefore, possess a differential coefficient at the origin. The  $\delta$ -function is, therefore, a myth. Dirac himself has admitted that it is an improper function, but that has not prevented an extensive use being made of the function in Quantum Mechanics.

We have come across only one specific example of such a function defined as follows (Jeans, II, p. 519):—

$$F(x) = Lt \quad \frac{c}{\pi(x^2 + c^2)}, \quad c \rightarrow 0$$



This equals zero when  $x \neq 0$ . For the integral property, presumably the mathematics is as follows:—

$$\begin{aligned} \int_{-\infty}^{\infty} F(x) dx &= \int_{-\infty}^{\infty} \lim_{c \rightarrow 0} \frac{Lt}{\pi(x^2 + c^2)} \frac{cdx}{c} = \lim_{c \rightarrow 0} \frac{Lt}{\pi} \int_{-\infty}^{\infty} \frac{dx}{c^2 + x^2} \\ &= \lim_{c \rightarrow 0} \frac{Lt}{\pi} \left[ \tan^{-1} \frac{x}{c} \right]_{-\infty}^{\infty} = 1. \end{aligned}$$

This process involves an interchange of limiting operations which is not usually permissible.

A few words may be added about the derivation of Schrödinger's equation. He derived his equation by a process of extremalisation of the expression for energy for which no justification could be given. The present tendency is to assume de Broglie's principle of equivalence of matter and waves and deduce partly by logic and partly by intuition, the well-known wave equation (Richtmyer and Kennard, p. 259, *et seq.*). Here again the continually shifting ground points to the weakness of the logical foundation.

12. Among the mathematical difficulties and anomalies, a few are mentioned below. Undoubtedly there are others which must have struck critically discriminating mathematicians.

(i) Non-commutative multiplication which is one of the corner-stones of Quantum Mechanics, does not present any difficulty to the mathematician. But when exponentials are introduced to represent dynamical variables (Dirac's *q*-numbers) non-commutative multiplication must lead to non-commutative addition (Dirac, p. 136, *et seq.*).



If  $e^a e^b \neq e^b e^a$ , then  $a + b \neq b + a$ .

Logically, therefore, we must take into account the order in the sum of two dynamical variables. This would create immense difficulties in manipulation, leading to a morass, in which further progress is impossible.

(ii) In Lande's expression for the anomalous Zeeman effect, it is usual to substitute expression like  $j(j+1)$  for  $j^2$  (Sommerfeld, p. 629). This is the sort of mathematics which is common among schoolboys wishing to get the answer correct and hoping that the trick would miss the eye of the teacher.

(iii) Consider again Dirac's well-known relativistic equation

$$H \equiv -c(\alpha_x p_x + \alpha_y p_y + \alpha_z p_z) - \rho_z m c^2 \quad (\text{Dirac, p. 260})$$

of the motion of a free electron. We get easily the surprising result

$$\dot{x} = [x, H] = -c \alpha_x.$$

It has as eigenvalues  $\pm c$  corresponding to the eigenvalues  $\pm 1$  of  $\alpha_x$ . As  $\dot{y}$  and  $\dot{z}$  are similar, we can conclude that a *measurement* of a component of the velocity of a free electron is *certain* (italics are ours) to lead to the result  $\pm c$ . This conclusion is easily seen to hold also when there is a field present.

Dirac has taken great pains to explain away this astounding result. He asserts that the contradiction is not real though, since the theoretical velocity in the above conclusion is the velocity at one instant of time while observed velocities are always average velocities through appreciable time intervals.



He shows that "the  $x$ -component of velocity  $-ca_x$  consists of two parts  $c^2 p_x H^{-1}$ , connected with the momentum by the classical relativistic formula, and an oscillatory

part 
$$-\frac{1}{2}ic\hbar\alpha_x^0 e^{-2iHt/\hbar} H^{-1}$$

whose frequency is high, being  $2H/\hbar$  which is at least  $2mc^2/\hbar$ . . . . . The oscillatory part secures that the instantaneous value of  $x$  shall have the eigenvalues  $\pm c$ ."

It is difficult to meet any argument based on the uncertainty principle. But we can look upon the result from another angle. If the experimental error in the measurement of the velocity of an electron be of the order of millions per cent., the whole structure of experimental Physics breaks down. Also if the velocity of an electron is indeterminate to the extent of the velocity of light, its mass should also be indeterminate without limit if the Relativity theory is not to be given up.

Again, 
$$\dot{x} : \dot{y} : \dot{z} = \alpha_x : \alpha_y : \alpha_z$$

or, the velocity of the electron is in the same direction as the axis of spin, whatever the origin may be. Perhaps we shall be told that the  $\alpha$ 's are not the spin components in the definite sense of classical mechanics. But what exactly do they stand for?

(iv) Dirac often speaks of a polarised photon. What exactly does it mean if it be conceived as a particle of energy? It must be a structure of electromagnetic waves before the word *polarised* may be applied to it. This is exactly the basis of the present thesis.

(v) The Bohr theory of emission states that if  $W_l$ ,  $W_m$  and  $W_n$  be three consecutive energy levels, the relation



with the frequency of the corresponding jumps are

$$W_l - W_m = h\nu_{lm}$$

$$W_m - W_n = h\nu_{mn}.$$

The expressions on the left hand side are additive while those on the right hand side are not (*i.e.*, not physically). If the electron makes the level  $W_m$  a halting station, two separate radiations of frequencies  $\nu_{lm}$  and  $\nu_{mn}$  are emitted. But if there be no halt at the intermediate level, only one radiation of frequency  $\nu_{ln}$  is emitted. Now the halt may be as short as one likes, theoretically at least, and there is no discontinuity between a finite halt and no halt. But physically  $\nu_{lm} + \nu_{mn} \neq \nu_{ln}$ . Red light superposed on green light does not make violet light.

A word of protest must be raised against the use of Newtonian ideas in theories of matter and electricity. Throughout the entire range of wave mechanics runs the basic idea of Newtonian mechanics mainly by way of Hamilton's canonical equations. This is open to serious objections. The concentration of finite mass at points or finite volumes is the foundation of Newtonian mechanics in which mass and energy are absolutely separate entities. In the electromagnetic theory, on the other hand, mass and energy are equivalent and they are spread out over the entire space. The only equations applicable are the Maxwell equations and even these require modification.

A curious feature of the new Physics calls for a word of comment. After some property has been deduced on certain assumptions, they are altered without any compunction that the previous arguments may thereby be



invalidated. One such example is the quantum numbers which must be integers in order that the solutions of the wave equation may be uniform, finite and continuous. But subsequently half odd integers are introduced as quantum numbers thus neutralising the previous work.



## CHAPTER II

### WHAT IS MATTER

1. What is matter? Modern Physics answers that it is made up of the fundamental particles, electrons, protons and neutrons. The two latter form the constituents of the nucleus held together by forces, believed to be 'exchange forces'. These possess the very necessary characteristic of saturation, but Tamm and Ivanenko have calculated that in order to be effective the distance between the elementary particles must be of the order  $10^{-17}$ , a value much smaller than the ordinarily accepted nuclear dimensions (NATURE 133, 1934). But about the structure of the fundamental particles modern Physics has nothing to say.

In our college days, we looked upon the elementary particles as something like billiard balls with a coat of paint, and even now we have not gone much further in our search for the objective reality. Only the balls ought to blow up, but for some reason they do not. They ought to possess infinite energy, but obviously cannot. Modern Physics has given up the search and is prepared to accept the fundamental properties of matter as postulates without concerning itself with the problem of the existence of some fundamental unity underneath. In fact, the primary object of science, to discover unity in the midst of diversity and to reduce the number of independent fundamental



concepts to the minimum number has been given up. Science has become in essentials, merely descriptive—that too in symbols only. Dirac maintains that it is no business of Quantum Mechanics to provide a picture. Its object, he says, is the formulation of laws governing phenomena and the application of these laws to the discovery of new phenomena. We have no quarrel with Dirac's enunciation of the object of science but consider a physical picture essential so that the theory can be checked at any stage where experimentation is possible.

2. To a classical physicist, any satisfactory theory of matter should solve the following fundamental problems:—

(i) identification of matter with energy, (ii) the de Broglie theory of equivalence of matter and waves, experimentally verified by Thomson, (iii) Coulomb's law, (iv) gravitation, *i.e.*, attraction at certain distances and repulsion at smaller, (v) Relativity variation of mass, (vi) a rational explanation of charge. Let us see how modern Physics faces these problems.

3. For the last half a century or so, the principle of identity of matter with energy has been generally recognised, but with a certain amount of mental reservation, hedged in with a good many 'yes, buts'. Intimately connected with this principle is the question of radiated solar energy. Until a few years ago, the conversion of matter into energy was the only theory existent. The following extract from Jeans (I, p. 68) gives a non-mathematical explanation:—

" This general evidence suggests that the sun has been pouring away mass in the form of radiation at a rate of

535.1

Se 474

T3CU 2786



250 million tons a minute for a period of some millions of millions of years. Detailed calculation shows that the new-born sun must have had many times the mass of the present sun in conformity with the general fact of observation that young stars are many times more massive than old stars. In what form could it store all the mass which has since disappeared in the form of radiation?

“The rest-mass of an electron or other charged particle is generally enormously greater than its energy-mass, the latter assuming its greatest importance at high temperatures. Now the temperature at the centre of the sun is about 50,000,000 degrees, and even here the rest-mass accounts for all but one part in 200,000 of the total mass.

“It is improbable that the new-born sun can have been much hotter than this, so that it seems likely that the greater part of the mass of the *primaeval* sun must have contained many more electrons and protons and therefore, many more atoms than now. These atoms can only have disappeared in one way: they must have been annihilated, and their mass must be represented by the mass of the radiation which the sun has emitted in its long life of millions and millions of years. Modern Physics gives no idea how the transformation takes place.” It is a mystic phenomenon—one of the mysteries of the mysterious universe.

At present the theory of Bethe—radiation of surplus energy of four Hydrogen atoms combining to form a Helium atom—has come to the front. Even then the conversion of the surplus mass into radiated energy is an important part of the theory.

GS 2109



4. It is difficult to realise the objective reality behind the de Broglie waves of Wave Mechanics. Jeans analyses the difficulty as follows (I, pp. 121-23):—

“ Most physicists would, I think, agree that the seven-dimensional space in which the wave mechanics pictures the meeting of two electrons is purely fictitious, in which case the waves which accompany the electrons must also be regarded as fictitious. Thus Professor Schrödinger, writing on the seven dimensional space says that although it has quite a definite physical meaning, it cannot very well be said to ‘ exist ’, hence a wave motion in this space cannot be said to ‘ exist ’ in the ordinary sense of the word either. It is merely an adequate mathematical description of what happens. It may be that also in the case of one single electron, the wave motion must not be taken to exist in too literal a sense, although the configuration-space happens to coincide with ordinary space in this particularly simple case.

“ Yet it is hard to see how we can attribute a lower degree of reality to the one set of waves than to the other: it is absurd to say that the waves of one electron are real, while those of two electrons are fictitious. . . . . Some physicists meet this situation by regarding the electron-waves as waves of probability. When we speak of a tidal wave we mean a material wave of water which wets everything in its path. When we speak of a heat wave, we mean something, although not material, warms up everything in its path. But when the evening papers speak of a suicide wave they do not mean that each person in the path of the wave will commit suicide: they merely



mean that the likelihood of his doing so is increased . . . . . The waves which represent an electron in the wave-mechanics may, it is suggested, be probability waves, whose intensity at any point measures the probability of the electron being at that point.

“ Thus at each point on Prof. Thomson's plate the wave intensity measures the probability that a single diffracted electron would hit the plate at that spot. When a whole crowd of electrons is diffracted, the total number which hit any spot is, of course, proportional to the probability of each individual hitting the spot, so that the darkening of the plate gives a measure of the probability per electron.

“ This view has the great merit that it enables the electrons to preserve their identity. If the electron-waves are true material waves, each system of waves would probably be dispersed by the experiment, so that no electrified particles would survive as such in any diffracted beam. Indeed, any encounter with matter would break up electrons, which could not be regarded as permanent structures. Actually, of course, it is the shower of electrons, rather than the individual, that is diffracted; the individual electrons move as particles and retain their identity as such.

“ All this is in accordance with Heisenberg's uncertainty principle, which makes it impossible ever to say: an electron is here, at this precise spot, and is moving at just so many miles an hour; we can only speak in terms of probabilities.”

This view is perfectly understandable and would have been satisfactory but for the objection that it offers no



explanation of the fact that the diffraction pattern is the same as would be obtained on the Debye-Scherrer theory taking the length of the waves as that given by the de Broglie theory. The problem must, therefore, be treated as an open one.

5. Coulomb's law and the Relativity variation of mass are accepted by modern Physics as postulates, no explanation being either necessary or available. It would be interesting to make a list of all the postulates and principles of modern Physics. It is sure to be an imposing one.

Gravitation is treated as something independent of all other manifestations of matter. The connecting link is furnished by the Einstein's principle that a particle of rest-mass  $m$  is equivalent to energy  $mc^2$  and the deviation of the path of light when passing close to the sun, which is one of the crucial tests of the Relativity theory.

6. Again the hydrodynamical idea of group velocity finds frequent mention in, Quantum Mechanics, when something of a permanent nature is sought to be created out of waves. As an example may be cited the attempt at constructing an electron by combining solutions of the Schrödinger equation. The mathematics in hydrodynamical language is as follows (Lamb, p. 360):—

Let two progressive waves  $a \sin(kx - \sigma t)$ ,  $a \sin(k'x - \sigma't)$  of the same amplitude, but of slightly different wave lengths be superposed. Then the resultant displacement is

$$a \{ \sin(kx - \sigma t) + \sin(k'x - \sigma't) \}$$

$$= 2a \cos \left\{ \frac{1}{2}(k - k')x - \frac{1}{2}(\sigma - \sigma')t \right\} \sin \left\{ \frac{1}{2}(k + k')x - \frac{1}{2}(\sigma + \sigma')t \right\}.$$



If  $k$  and  $k'$  be very nearly equal, the cosine in this expression varies very slowly with  $x$ ; so that the wave-profile at any instant has the form of a curve of sines in which the amplitude alternates gradually between the values 0 and  $2a$ . The surface, therefore, presents the appearance of a series of groups of waves separated at equal intervals by bands of nearly smooth water. The motion of each group is then sensibly independent of the presence of others. Since the distance between the successive groups is  $2\pi/(k-k')$  and the time taken by the system in shifting through the space is  $2\pi/(\sigma-\sigma')$  the group velocity

$$U = (\sigma - \sigma') / (k - k'),$$

or, ultimately  $U = d\sigma / dk$ .

In terms of the wave length

$$U = \frac{d(kw)}{dk} = w - \lambda \frac{dw}{d\lambda},$$

where  $w$  is the wave velocity. It is obvious that unless  $w$  is a function of  $\lambda$  the wave length, or the medium is dispersive, the group velocity is the same as the wave velocity.

For light waves the difference between the two velocities is, in general, very small, so that we can practically neglect it. It is quite otherwise with de Broglie's material waves. It can be shown (Frenkel I, p. 29) that the group velocity of material waves coincides with the velocity of the corresponding particles.

7. The above analysis has been modified with the help of Fourier's theorem so as to make the wave group short



and sharp. Here is the mathematical working (Kemble, p. 37):—

“ We define  $\Psi(x, t)$  by the equation

$$\Psi(x, t) \equiv \int_0^\infty G(\sigma) e^{2\pi i(x\sigma - vt)} d\sigma,$$

where  $G(\sigma)$  is assumed to satisfy the Dirichlet condition (for Fourier's integral theorems). Differentiation under the integral sign shows that  $\Psi(x, t)$  is a solution of the equation

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{4\pi\mu i}{h} \frac{\partial \Psi}{\partial t} = 0 \quad \text{if} \quad \frac{v}{h} = \frac{\sigma^2}{2\mu}.$$

(Schrödinger equation in one dimension in space under no forces; the potential energy being a constant has been taken to be zero.  $\mu$  is the mass of the electron.)

“ . . . Let us next consider a special case in which  $G(\sigma)$  is a function having a maximum at the point  $\sigma = \sigma_0$  and approaching zero monotonically as  $|\sigma - \sigma_0|$  increases. We shall suppose that  $G$  is sensibly equal to zero outside of a certain small interval  $M$  containing the point  $\sigma = \sigma_0$ . The wave function  $\Psi$  can then be described as a wave packet, since it is composed of a group or packet of monochromatic progressive waves all of which have approximately the same wave number  $\sigma$  and the same direction of motion.

“ In order to study the behaviour of such a packet it is convenient to resolve  $\Psi$  into its real and imaginary parts  $\Psi_r$  and  $\Psi_i$  respectively. Let  $G(\sigma)$  be real and positive. If we denote the phase angle  $2\pi(x\sigma - vt)$  by  $\phi$  we have



$$\Psi_r = \int_{-\infty}^{\infty} G(\sigma) \cos \phi \, d\sigma, \quad \Psi_t = \int_{-\infty}^{\infty} G(\sigma) \sin \phi \, d\sigma.$$

In evaluating these integrals  $x$  and  $t$  are to be treated as fixed parameters while  $v$  is a function of the variable of integration  $\sigma$ ." The group velocity is easily deduced in the form

$$\left( \frac{dv}{d\sigma} \right)_{\sigma_0} = w - \lambda \frac{dw}{d\lambda}.$$

Let us consider the expression a little more closely.  $G(\sigma)=0$  when  $|\sigma - \sigma_0| > M/2$ ,  $M$  being small, so that though the two definite integrals have been taken between the limits  $\pm \infty$  they are really between  $\sigma_0 - M/2$  and  $\sigma_0 + M/2$ . At any instant of time,  $t_0$ , the two integrands have a periodic distribution along the  $x$ -axis, *i.e.*, whatever the shape of the curve  $G(\sigma)$  may be, it is repeated at intervals of  $1/\sigma_0$  along the  $x$ -axis. Integration with respect to  $\sigma$  cannot take away this property of periodicity along the  $x$ -axis. So whatever physical entity the wave packet may represent, it must form a procession. So if the wave packet is taken to represent an electron, the logical conclusion would be that electrons can only move in an unending procession.

8. The difficulties of the wave packet theories find expression in the following extract from Kemble (pp. 10-12):—

"The essential requirements to be imposed upon a wave disturbance in order that it shall constitute a wave packet are that (a) it shall occupy a small volume, (b) it shall travel with definite speed, and (c) it shall travel in a definite



direction. These requirements are, to a certain extent, mutually contradictory, as may be proved theoretically or demonstrated by appropriate experiment. Thus, in the case of an infinite train of plane waves the direction of motion is perfectly definite, but the disturbance is not localized at all. . . . Similarly, we may localize the wave train longitudinally if we reduce it to finite length in any manner. If the train is monochromatic and long enough to include many waves, it will travel with a fairly definite speed even in a dispersive medium. . . . However, if the train is made so short that it contains only a few waves or perhaps only a fraction of a wave, the group does not hang together, but spreads out longitudinally as it proceeds as if composed of dissimilar elements travelling at different rates of speed. Consequently, this second kind of localization of a wave disturbance must not be carried too far if it is not to conflict with the second requirement for a wave packet." Kemble thinks that by a suitable compromise, we can devise wave disturbances appropriate to a dispersive medium which satisfy all the three requirements of a wave packet.

These wave packets are sometimes meant to be material, *i.e.*, they are supposed to represent the elementary particles of matter. But it is more usual now to regard them as probability waves, *i.e.*, they represent the probability of any particle being found near about the particular place. It seems that Kemble regards the packets in the former sense.

9. In spite of the optimism of Kemble that a suitable compromise is possible among the contradictory require



ments of the packet idea, there are physicists who do not share his conviction. Here is an extract from Frenkel (p. 31):—

“ To interpret the spatial extension (radius) usually attributed to electrons (on the basis of the electro-magnetic theory of mass) Schrödinger . . . . . preferred to represent electrons by wave packets of finite dimensions. The latter point of view is, however, contradicted by the fact that wave packets do not preserve the same dimensions, but spread out in space as time goes on, whereas the dimensions of the particles they represent remain constant.

“ But even apart from this, the conception of particles as wave packets contradicts the facts of observation on the interference and diffraction of the corresponding rays. If we imagine a bundle of cathode rays to be a collection of wave packets, each wave packet corresponding to a single electron, then the scattering of these rays at a diffraction grating would not produce the usual interference pattern similar to that which is observed in the case of homogeneous light waves. In fact, to explain this interference pattern by the wave theory we must assume that the incident rays form a sinusoidal train of waves with a constant amplitude through space (disregarding of course the finite cross-section and divergence of the bundle of rays).

“ Thus the representation of the individual electrons in a beam of cathode rays by wave packets is quite unable to explain diffraction and interference phenomena. The influence of the grating would manifest itself in this case in the reflection and eventually in the spreading of the





separate wave packets, more or less in accordance with the corpuscular theory, and no trace of the actually observed interference and diffraction phenomena will be displayed.

“ Discontinuous elementary actions of light or material rays (such as the photo-electric effect), . . . . . cannot be explained by a corpuscular structure of the corresponding wave field. They must also occur with a completely homogeneous structure—or rather a complete absence of structure—of this field.

“ We must see that the reduction of particles to waves made so tempting by the coincidence of particle velocity with group velocity is impossible and that this coincidence must have an entirely different physical meaning.”

On this last point, we may observe that the velocity of light is mean proportional between the group velocity and the phase velocity. So the phase velocity is greater than the velocity of light, a contingency which is unthinkable to a relativist. Consequently, if the Relativity theory is not to be given up, the phase velocity can have no physical significance and consequently the group velocity can have no physical import either.

Lastly, the wave packet theory does not offer any explanation of the Relativity variation of mass. On the contrary, Doppler's principle would indicate quite a wrong expression  $\sqrt{(1-v/c)/(1+v/c)}$  or,  $\sqrt{(1+v/c)/(1-v/c)}$  instead of the well-known factor  $1/(1-v^2/c^2)^{1/2}$  for the change of mass.

10. As regards the second alternative of the wave packets representing the probability of a particle being



found near about a particular place, it may be pointed out that the tendency of the packets to spread out persists and however sharply defined the packet may be initially, it will be blurred in the course of time and the probability packet will cease to have any meaning.

The subject has the tendency to grow more and more vague. Says Jeans (I, p. 124): "Heisenberg and Bohr have suggested that electron waves must be regarded merely as a sort of symbolic representation of our knowledge as to the probable state and position of an electron. If so, they change as our knowledge changes, and so becomes largely subjective. Thus we need hardly think of the waves as being located in space and time at all; they are mere visualisations of a mathematical formula of an undulatory but wholly abstract nature.

"A still more drastic possibility, again arising out of a suggestion made by Bohr, is that the minutest phenomena of nature do not admit of representation in the space-time framework at all. On this view the four-dimensional continuum of the theory of Relativity is adequate only for some of the phenomena of nature, these including large-scale phenomena and radiation in free space; other phenomena can only be represented by going outside the continuum. We have, for instance, already tentatively pictured consciousness as something outside the continuum and have seen how the meeting of two electrons can most simply be pictured in seven dimensions. It is conceivable that happenings entirely outside the continuum determine what we describe as the 'course of events' inside the continuum, and that the apparent indeterminacy of nature



may arise from our trying to force happenings which occur in many dimensions into a smaller number of dimensions." If this represents a fair concensus of opinion, physics is not very far from the conclusion of one school of Hindu philosophy that matter is an illusion, being only a manifestation of the Supreme Consciousness.

Again, the probability of an event offers no clue to the nature or character of the actors in the drama. So, even if the probability wave packet be a fact, we are no wiser than before about the nature and character of the particle. The probability of this booklet being on the table of a scientist is no indication of the correctness or the falsity of the thesis! **As far as any indication about the physical structure of the particle is concerned, the probability theory is mere empty verbiage. It attempts an answer to the query "where is matter?" and not "what is matter?"**

11. At present are known about twenty-five or thirty so-called fundamental particles which, in spite of wide difference of mass, possess the remarkable property that the charge is either  $\pm e$  or zero. There is also the neutrino which has been conceived by Fermi only to explain the residual property with no possibility of actual demonstration! It is a particle too small for experimental verification. Theoretical physics must have come to a pretty pass to have to swallow such a big pill and that too in an age that scorns concepts and places its faith on observation alone.

12. A few words may not be out of place in criticism of the doctrine of chance, which lays down that things happen not because they must, but because they are more



likely than any other contingency. A universal illustration of the doctrine is the spinning of a coin. In fact, the probability of heads and tails turning up in equal proportions in a large number of tosses is taken as axiomatic in the new theory. It seems to be the most self-evident truth much more fundamental than many of the axioms hitherto accepted without a shadow of doubt.

Jeans says: "We can illustrate the concept by an analogous situation in a large-scale world. If we spin a half-penny, *nothing within our knowledge* (italics are ours) may be able to decide whether it will come down heads or tails, yet if we throw up a million tons of half-pence, we know there will be 500,000 tons of heads and 500,000 tons of tails. The experiment may be repeated indefinitely and will always give the same result" (I, p. 28).

But the spin of a coin is a simple case of motion of a rigid body (if it is rigid) and any mathematician can predict the result of a toss provided the initial conditions, the initial position, impulse, etc. are known. The uncertain element is the lack of definite information and the difficulty of ascertaining them when a coin is tossed with the fingers. But certainly it is not beyond the human ingenuity to devise a machine which could spin a coin head up every time, if any one could be found to spend his energy on anything so futile!

A classicist would reply that Jeans' contention would hold only if the throws are *absolutely random* and challenge the assertion that we cannot know the outcome of the throw. But a number of throws absolutely at random is just as inconceivable as a number of *identical* throws.



Both are circumscribed by the weakness of the human material and mind. To a classical mathematician, statistical mechanics gives only an approximate average solution of a hopelessly complicated problem. When we have a large number of unknowns and a large number of unknown relations, statistical mechanics gives a reliable guide to extrapolation. But it is no substitute for causality.

13. The earliest observed manifestation of electricity in nature was lightning and in the laboratory, the electric charge. In the days of Benjamin Franklin (1706-1790), there was a big controversy about the nature of electricity. There was no doubt whatever that it was a fluid, the controversy was whether it was a single fluid or two fluids. Even in the latter part of the nineteenth century, the Maxwell displacement was essentially a confirmation of the fluid theory. The discovery of the electron and the proton localised the fluid on the surface of the particles, thus creating the problem of stability of these charges. Modern physics has accepted the fluid theory by implication, leaving the question of stability unsolved.

It is doubtful if there is any living physicist who believes in it. Yet nobody seems to mind. It is obvious that such an elementary manifestation of electricity cannot be ignored in any theory without grave risk of collapse due to defective foundations.

The very fact that a proton possessing 1840 times the mass of the electron has the same charge but of opposite sign, points to the conclusion that the charge is not a primary property but a secondary one. Besides these are known at present twenty-five or thirty so-called funda-



mental particles including the neutron and the recently discovered anti-neutron. There are different varieties of mesons possessing different masses and energies. But the significant property is that the charge is either  $\pm e$  or zero. This clamours for an explanation.

14. As the conclusion to all this discussion, it may be stated without any fear of contradiction that a self-consistent theory about the structure of the fundamental particle does not exist. The attempts that have been made have met with scanty success. There is no room for satisfaction with the existing state of Physics, much less for self-complacency. The mental attitude of physicists is reminiscent of the dogma of the scholastics—'I believe in order to understand.'

---



## CHAPTER III

### A NEW THEORY OF LIGHT

1. The starting point is the Maxwell's electromagnetic equations in free space, *i.e.*, without any charge either at rest or in motion. Let  $E$  represent the electric vector with the components  $E_x, E_y, E_z$  and  $H$  the magnetic vector with the components  $H_x, H_y, H_z$ . Then the equations may be written as

$$\begin{aligned}\text{curl } E + \frac{1}{c} \dot{H} &= 0, \\ \text{div } H &= 0, \\ \text{curl } H - \frac{1}{c} \dot{E} &= 0, \\ \text{div } E &= 0. \quad \dots (1)\end{aligned}$$

It is easily seen that the electric and magnetic components severally satisfy the wave equation

$$\nabla^2 F - \frac{1}{c^2} \frac{d^2 F}{dt^2} = 0. \quad \dots (2)$$

Consider now a rectilinear progressive wave along the  $x$ -axis. We proceed to show that if it presents the Doppler effect and is subject to the Principle of Relativity, the electric and magnetic components must be given by equations of the form

$$\begin{aligned}E_y = H_z &= A \sin k(x - ct)/(x - ct), \\ E_x = E_z = H_x = H_y &= 0. \quad \dots (3)\end{aligned}$$



The general solution of equations (1, 2) in one dimension of space is given by

$$E_y = H_z = f(x-ct) \quad \dots (4)$$

the other electric and magnetic components being zero. If this has to present the Doppler effect,  $f$  must involve a factor  $\sin k(x-ct)$ . To make the sinusoidal form explicit, we write

$$E_y = H_z = \phi(x-ct) \sin k(x-ct).$$

The Lorentz transformation for an observer  $O'$  (coordinates  $x', y', z', t'$ ) moving with velocity  $v$  in the direction of the  $x$ -axis is

$$x = \beta(x' + vt'), \quad y = y', \quad z = z', \quad t = \beta(t' + vx'/c^2),$$

$$\beta = (1 - v^2/c^2)^{-\frac{1}{2}}.$$

We have then

$$x - ct = \beta(1 - v/c) (x' - ct'). \quad \dots (5)$$

On application of this transformation, the factor  $\sin k(x-ct)$  becomes  $\sin k'(x'-ct')$ , where

$$k' = k\beta(1 - v/c) = k\sqrt{(1 - v/c)/(1 + v/c)}.$$

Introducing the frequency  $\nu = kc/2\pi$ , we have

$$\nu' = \nu\sqrt{(1 - v/c)/(1 + v/c)}.$$

This is, of course, the Doppler effect.

It is well known after Larmor and Lorentz that the Maxwell equations are conserved under the transformation provided we write

$$\mathcal{E}_{x'} = E_x = 0$$

$$\mathcal{E}_{y'} = \beta(E_y - v/c H_z) = \beta(1 - v/c)f(x-ct)$$



$$\mathcal{E}_{z'} = \beta(E_z + v/c H_y) = 0$$

$$\mathcal{H}_{x'} = H_x = 0$$

$$\mathcal{H}_{y'} = \beta(H_y + v/c E_z) = 0$$

$$\mathcal{H}_{z'} = \beta(H_z - v/c E_y) = \beta(1 - v/c)f(x - ct). \quad (6)$$

(Tolman, p. 87 with slight change of notation).

But  $\mathcal{E}_{x'}$ ,  $\mathcal{E}_{y'}$ ,  $\mathcal{E}_{z'}$ ,  $\mathcal{H}_{x'}$ ,  $\mathcal{H}_{y'}$ ,  $\mathcal{H}_{z'}$  are not necessarily the electromagnetic vector at the point  $x'$ ,  $y'$ ,  $z'$ ,  $t'$ . Since the Maxwell equations are homogeneous, we may write the actual electromagnetic vector

$$E_{x'}, E_{y'}, E_{z'}, H_{x'}, H_{y'}, H_{z'}$$

$$\text{as } \lambda \mathcal{E}_{x'}, \lambda \mathcal{E}_{y'}, \lambda \mathcal{E}_{z'}, \lambda \mathcal{H}_{x'}, \lambda \mathcal{H}_{y'}, \lambda \mathcal{H}_{z'},$$

where  $\lambda$  is a constant—either absolute or a function of  $v$ .

The first alternative is barred by the condition that  $E_{y'}$ ,  $H_{z'}$  must reduce to  $E_y$ ,  $H_z$  when  $v = 0$ . Therefore,  $\lambda$  can only be a function of the type  $\psi(v/c)$  which reduces to 1 when  $v = 0$ .

We may then write

$$E_{y'} = \beta(1 - v/c)\psi(v/c)E_y$$

$$H_{z'} = \beta(1 - v/c)\psi(v/c)H_z \quad \dots \quad (7)$$

the other components being zero. The factor  $\beta(1 - v/c)$  may be absorbed in  $\psi(v/c)$  since it also becomes 1 when  $v = 0$ . We have then

$$E_{y'} = \psi(v/c)E_y, \quad H_{z'} = \psi(v/c)H_z,$$

$E_{y'}$ ,  $H_{z'}$  representing the electromagnetic vector for the observer  $O'$  must be the same function of  $x'$ ,  $t'$  as  $E_y$ ,  $H_z$  are of  $x$ ,  $t$ , since  $E_{y'}$ ,  $H_{z'}$  must reduce to  $E_y$ ,  $H_z$  when  $v = 0$  and  $x' = x$ ,  $t' = t$ . It will be remembered that the Principle of Relativity postulates that nature has no preference for one observer over another.  $E_{y'}$  as a



function of  $x'$ ,  $t'$  is, therefore, independent of the observer  $O$ , though it reduces to  $E_y$  when  $v=0$ . Ignoring the sine factor for the present, we thus have

$$E_y = \phi(x - ct)$$

$$E_{y'} = \psi(v/c)E_y = \phi(x' - ct').$$

We impose on the equations another Lorentz transformation for an observer  $O''$  (coordinates  $x''$ ,  $y''$ ,  $z''$ ,  $t''$ ) moving with velocity  $v'$  relative to  $O'$ . Then by the usual formula of Relativity Kinematics, the velocity of  $O''$  relative to  $O$  is

$$w = \frac{v + v'}{1 + vv'/c^2}.$$

If  $E_{y''}$ ,  $H_{z''}$  represent the electromagnetic vector for the observer  $O''$

$$\begin{aligned} E_{y''} &= \psi(v/c)\psi(v'/c)\phi(x - ct) \\ &= \psi(w/c)\phi(x - ct) \end{aligned} \quad \dots (9)$$

by direct transformation.

On comparison we get

$$\psi(v/c)\psi(v'/c) = \psi\left(\frac{v + v'/c}{1 + vv'/c^2}\right)$$

for the determination of  $\psi$ .

Putting  $\psi = e^{\chi}$

we have  $\chi(v/c) + \chi(v'/c) = \chi\left(\frac{v + v'/c}{1 + vv'/c^2}\right).$



Put again  $v/c = \tanh \theta$ ,  $v'/c = \tanh \theta'$ , then

$$\frac{v + v'/c}{1 + vv'/c^2} = \tanh(\theta + \theta')$$

So  $\chi(\tanh \theta) + \chi(\tanh \theta') = \chi(\tanh(\theta + \theta'))$ .

The general solution is  $\chi = n \tanh^{-1}$ ,  $n$  being any number positive or negative, integral or fractional.

Again, 
$$\frac{v}{c} = \frac{e^\theta - e^{-\theta}}{e^\theta + e^{-\theta}}$$

or, 
$$\frac{1 - v/c}{1 + v/c} = e^{-2\theta}$$

or, 
$$e^{-\theta} = \sqrt{(1 - v/c)/(1 + v/c)} = \beta(1 - v/c).$$

We have, therefore,  $\chi(v/c) = n\theta$

or,  $\psi(v/c) = e^{n\theta} = \{\beta(1 - v/c)\}^{-n}$ .

We get, therefore,

$$E_{y'} = \phi(x' - ct') = \{\beta(1 - v/c)\}^{-n} \phi(x - ct).$$

Also,  $x - ct = \beta(1 - v/c)(x' - ct')$ .

Eliminating  $\beta(1 - v/c)$  we have

$$(x' - ct')^{-n} \phi(x' - ct') = (x - ct)^{-n} \phi(x - ct).$$

The left hand side is a function of  $x'$ ,  $t'$  and the right hand side of  $x$ ,  $t$ . We must have, therefore, each side a universal constant  $A$ .

We get  $\phi(x - ct) = A(x - ct)^n$

or,  $E_y = A(x - ct)^n \sin k(x - ct) \dots (10)$

on restoring the sine factor.



There are only two values of  $n$  which does not make  $E_y$  infinite at some point or other. The first is  $n=0$ , giving

$$E_y = H_z = A \sin k(x - ct).$$

This is the basis of the old classical theory which has been found inadequate. The other value is  $n = -1$ , giving

$$E_y = H_z = A \sin k(x - ct)/(x - ct) \quad \dots (11)$$

which is taken as the basis of our theory. Fractional values of  $n$  are inadmissible since they make the vector multiple-valued.

**The conclusion is, therefore, irresistible that if the Maxwell equations hold good as well as the special Relativity theory, a linear photon obeying Doppler's principle must have its electric and magnetic vectors perpendicular and in a plane perpendicular to the direction of propagation and in the form (11).** There is no way of avoiding this formula unless we are illogical enough to accept the premises and deny the conclusion.

2. It is usual, when considering any solution of the Maxwell equations to introduce the conception of a wave front (see, *e.g.*, Frenkel, pp. 15-16). This is a relic of Fresnel and Young's theory in which every point of the wave front was supposed to be a centre of disturbance and the wave front was supposed to move perpendicular to the ray. In the Maxwell theory the direction of propagation of the beam plays a fundamental part and the wave front is not only superfluous but misleading. If we take the Maxwell field  $E_y = H_z = f(x - ct)$  to cover the entire space, we lose sight of the individual beam which is our main concern. We get, on the other hand, a bundle of



parallel beams, in which no distinction is made between the rectilinear path of the beam and any other straight line in space parallel to it. From this standpoint, if  $E_y = H_z = f(x-ct)$  represents a wave along the  $x$ -axis, it can also very well represent a wave along the straight line  $y=l, z=k$ .

Consequently, we replace the expressions for the electromagnetic vector for a beam of light along the  $x$ -axis by

$$E_y = A e^{-(\lambda_1^2 y^2 + \lambda_2^2 z^2)} \sin k(x-ct)/(x-ct)$$

$$H_z = A e^{-(\lambda_1^2 y^2 + \lambda_2^2 z^2)} \sin k(x-ct)/(x-ct)$$

all the other components being zero. These satisfy the Maxwell equations along the path of the beam. We identify the photon with this type of electromagnetic wave and proceed to find an expression for the energy. The energy of the electric field at any instant may be written as (putting  $\xi = x-ct$ ),

$$\begin{aligned} W_e &= \frac{A^2}{8\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2(\lambda_1^2 y^2 + \lambda_2^2 z^2)} \frac{\sin^2 k\xi}{\xi^2} d\xi dy dz \\ &= \frac{A^2}{\pi} \int_0^{\infty} e^{-2\lambda_1^2 y^2} dy \int_0^{\infty} e^{-2\lambda_2^2 z^2} dz \int_0^{\infty} \frac{\sin^2 k\xi}{\xi^2} d\xi \\ &= \frac{A^2}{\pi} \cdot \frac{\pi}{8\lambda_1\lambda_2} \left[ -\frac{\sin^2 k\xi}{\xi} \Big|_0^{\infty} + k \int_0^{\infty} \frac{\sin 2k\xi}{\xi} d\xi \right] \\ &= \frac{A^2 k}{8\lambda_1\lambda_2} \int_0^{\infty} \frac{\sin 2k\xi}{\xi} d\xi = \frac{\pi A^2 k}{16\lambda_1\lambda_2} \end{aligned}$$

Similarly for the magnetic field.



Putting  $\lambda_1 \lambda_2 = \pi/2\mu^2$ , we get the total electromagnetic energy as  $W = \frac{1}{4} A^2 \mu^2 k$ ,  $\mu$  having the dimension of a length.

In the first edition, the electric vector was assumed to be confined to the  $xy$ -plane and the magnetic vector to the  $xz$ -plane. This assumption suffered from the drawback that the electric and the magnetic vectors were discontinuous in the directions perpendicular to these planes respectively and so not differentiable. It is easily seen that this is the limiting case when  $\lambda_1 \rightarrow 0$ ,  $\lambda_2 \rightarrow \infty$ . The electric vector is then zero at every point outside the  $xy$ -plane and the magnetic vector is zero at every point outside the  $xz$ -plane. It will be convenient to work on this hypothesis. We proceed to find an expression for the energy.

3. The energy of the electric field in a thin slab of thickness  $\delta$  along the  $z$ -axis and breadth  $l$  along the  $y$ -axis at any instant is given by

$$l\delta \int_{-\infty}^{\infty} E_y^2 / 8\pi dx.$$

To cover the entire  $xy$ -plane, we have to make  $\rightarrow \infty$  and  $\delta \rightarrow 0$ . We assume that the limit  $l\delta$  exists and is equal to  $\mu^2$ . The total energy, electric and magnetic, of the photon at any instant, is, therefore, given by (putting  $\xi = x - ct$ )

$$W = \frac{A^2 \mu^2}{4\pi} \int_{-\infty}^{\infty} \frac{\sin^2 k\xi}{\xi^2} d\xi = \frac{A^2 \mu^2}{2\pi} \int_0^{\infty} \frac{\sin^2 k\xi}{\xi^2} d\xi$$



$$\begin{aligned}
&= \frac{A^2 \mu^2}{2\pi} \left[ -\frac{\sin^2 k\xi}{\xi} \Big|_0^x + \int_0^x \frac{k \sin 2k\xi}{\xi} d\xi \right] \\
&= \frac{A^2 \mu^2 k}{2\pi} \int_0^x \frac{\sin 2k\xi}{\xi} d\xi = \frac{A^2 \mu^2 k}{4} \dots (12)
\end{aligned}$$

The factor  $\mu^2$ , therefore, comes in when the volume integral is replaced by a line integral,  $\mu$  having the dimension of a length.

4. Let us draw a graph of the function  $A \sin k\xi/\xi$ . It possesses a hump at  $\xi=0$  and has nodes at  $\pm\pi/k$ ,  $\pm 2\pi/k$ ,  $\pm 3\pi/k$ , .....

The amplitude at  $\xi=0$  is  $Ak$  and the successive maximum ordinates are approximately of magnitudes  $\frac{2}{3} Ak$ ,  $\frac{1}{5} Ak$ ,  $\frac{1}{11} Ak$ , ... and trail off to zero on either side. The hump, therefore, is about five times the maximum ordinate of the nearest wave. This particular solution, therefore, possesses the main characteristic of a pulse, namely, concentration of energy and impulse to a limited region of the wave. Since it possesses the velocity of light, an experiment on the distribution of energy and impulse in the different parts of the wave cannot be thought of.

We take the wave length  $\lambda=2\pi/k$ , the wave length of the numerator of the expression  $A \sin k(x-ct)/(x-ct)$ . The frequency  $\nu=kc/2\pi$  and the total energy

$$W = \frac{1}{4} A^2 \mu^2 k = h\nu, \text{ where } h = \pi A^2 \mu^2 / 2c.$$

To get an expression for the impulse, we note that the Maxwell equations being now valid only on the path of the beam. Poynting's theorem is invalidated. An expression may be derived as follows. Consider the energy enclosed



within a closed surface  $S$  in a vector field given by  $E=f(x-ct)$ . Since  $x$  and  $t$  enter only in the combination  $x-ct$ ,

$$\frac{\partial}{c\partial t} = -\frac{\partial}{\partial x}.$$

$$W_e = \frac{1}{8\pi} \iiint F dx dy dz, \text{ where } F = f^2.$$

$$\begin{aligned} \frac{\partial W_e}{c\partial t} &= -\frac{\partial W_e}{\partial x} = -\frac{1}{8\pi} \frac{\partial}{\partial x} \iiint F dx dy dz \\ &= -\frac{1}{8\pi} \iiint \frac{\partial F}{\partial x} dx dy dz = -\frac{1}{8\pi} \iint (F_{x_2} - F_{x_1}) dy dz \end{aligned}$$

integrating along a thin rod parallel to the  $x$ -axis,  $x_2$  and  $x_1$  being the  $x$ -coordinates of the ends of the rod,

$$= -\frac{1}{8\pi} \int l F dS$$

over the surface,  $l$  being the  $x$ -direction cosine of normal.

Thus the rate of increase of energy within the surface is equal to the rate of flow of  $cf^2/8\pi$  across and into the surface in the direction of the  $x$ -axis. The Poynting vector for the special case of a progressive electromagnetic wave

$$S_e = \frac{c}{8\pi} \frac{A^2 \sin^2 k\xi}{\xi^2}.$$

Similarly for the magnetic energy. The impulse is, therefore,

$$\frac{A^2}{4\pi c} \iiint \frac{\sin^2 k\xi}{\xi^2} d\xi dy dz = \frac{h\nu}{c}.$$



More simply, if we assume the postulate of the Relativity theory,

$$mc^2 = h\nu$$

the impulse

$$mc = h\nu/c.$$

Taking  $h = 6.5 \times 10^{-27}$ , we have  $A\mu = 1.1 \times 10^{-8}$ .

5. It is obvious that the expression

$$\frac{A \sin k\xi}{\xi} = A \int_0^k \cos k\xi dk$$

represents a packet of simple harmonic waves, but it shows only the length of the shortest wave in its composition. **It is extremely significant that the expression which has been derived from the Maxwell equations with an eye on the Relativity considerations alone, should stand for a packet of simple harmonic waves.** It is our view that the coincidence is not merely fortuitous but has a deep physical significance. It is our contention that the packet is more than a mere conglomeration of harmonic waves and possesses a distinct identity of its own.

The integral

$$A \int_0^{k_1} \cos k\xi dk$$

can, of course, be split up mathematically into

$$A \left\{ \int_0^{k_1} + \int_{k_1}^{k_2} \cos k\xi dk \right\} = A \left\{ \frac{\sin k_1\xi}{\xi} + \frac{\sin k_2\xi - \sin k_1\xi}{\xi} \right\}.$$

That this is also physically possible may be verified by calculating the energy of the second component

$$(\sin k_2\xi - \sin k_1\xi)/\xi.$$



Consider a photon given by the equation

$$E_{y1} = H_{z1} = A \sin k_1 \xi / \xi$$

and superpose on it another

$$E_{y2} = H_{z2} = -A \sin k_2 \xi / \xi$$

in the opposite phase. The total energy of the combination is given by the expression

$$\begin{aligned} W &= \frac{A^2 \mu^2}{4\pi} \int_{-\infty}^{\infty} \frac{(\sin k_1 \xi - \sin k_2 \xi)^2}{\xi^2} d\xi \\ &= \frac{A^2 \mu^2}{2\pi} \int_0^{\infty} \frac{(\sin^2 k_1 \xi + \sin^2 k_2 \xi - 2 \sin k_1 \xi \sin k_2 \xi)}{\xi^2} d\xi \\ &= \frac{A^2 \mu^2 (k_1 + k_2)}{4} + \frac{A^2 \mu^2}{2\pi} \int_0^{\infty} \frac{\cos (k_1 + k_2) \xi - \cos (k_1 - k_2) \xi}{\xi^2} d\xi \\ &= \frac{A^2 \mu^2 (k_1 + k_2)}{4} + \frac{A^2 \mu^2}{2\pi} \left[ -\frac{\cos (k_1 + k_2) \xi - \cos (k_1 - k_2) \xi}{\xi} \right]_0^{\infty} \\ &\quad - \int_0^{\infty} \frac{(k_1 + k_2) \sin (k_1 + k_2) \xi - (k_1 - k_2) \sin (k_1 - k_2) \xi}{\xi} d\xi \\ &= \frac{A^2 \mu^2 (k_1 + k_2)}{4} - \frac{A^2 \mu^2}{2\pi} \left\{ (k_1 + k_2) - (k_1 - k_2) \right\} \frac{\pi}{2} \\ &= \frac{A^2 \mu^2 (k_1 - k_2)}{4} = h(\nu_1 - \nu_2), \end{aligned}$$

where  $\nu_1 = k_1 c / 2\pi$ ,  $\nu_2 = k_2 c / 2\pi$ .

The energy of the combination, which may be called a partial photon is thus the difference of the energy of the two complete photons. It is easily verified that two



photons in the same phase cannot be superposed, for the energy of the combination

$E_{y1} = H_{z1} = A \sin k_1 \xi / \xi, \quad E_{y2} = H_{z2} = A \sin k_2 \xi / \xi$   
is obtained as

$$\frac{A^2 \mu^2 (k_1 + k_2)}{4} + \frac{A^2 \mu^2}{4} \{ (k_1 + k_2) - (k_1 - k_2) \} \neq h(\nu_1 + \nu_2),$$

which contradicts the Principle of Energy. This, of course, agrees with observation.

A photon of energy  $h\nu$  may, therefore, be split up into a photon of less energy  $h\nu_1$ , and a partial photon of energy  $h(\nu - \nu_1)$ , and the process of subdivision can be carried on without any limit. In particular if  $k - k_1$  is small and equal to  $\delta k$ , while  $\xi$  is not too large, the primary photon may be split up into a photon of slightly less energy  $h\nu_1$ , and a train of simple harmonic electromagnetic waves given by

$$E_y = H_z = A (\sin k\xi - \sin k_1\xi) / \xi = A\delta k \cos k\xi.$$

The latter is equivalent to a train of ether waves in the language of Physical Optics and may be identified with a beam of light. It presents the familiar phenomena of reflection, refraction, interference, diffraction etc., which are treated so simply and successfully in Physical Optics.

6. We have, therefore, two distinct entities, *viz.*, the complete photon given by

$$A \sin k\xi / \xi$$

and the partial photon

$$A (\sin k\xi - \sin k_1\xi) / \xi = A\delta k \cos k\xi$$



to explain the particle and wave properties of light. They possess the same wave length  $2\pi/k$  and are inextricably mixed up together. The difference is that while the complete photon possesses a finite energy  $h\nu$ , the partial photon possesses only infinitesimal energy proportional to  $\delta k$ . In experiments involving the energy and momentum such as the Compton effect, the photoelectric effect and the energy distribution in the spectrum, it is the complete photons of energy  $h\nu$  which preponderate. While in experiments on wave length the partial photons of small energy figure; complete photons of large energy, few in number, keep their individuality and are unaffected by the experiment and do not show. A partial photon

$$A \int_{k_i}^k \cos k\xi dk$$

on the other hand is easily resolved into the constituent waves by being passed through a prism, a complete photon

$$A \sin k\xi/\xi$$

behaving like a wave of length  $2\pi/k$  goes through unaffected. Curiously enough, this view finds a certain amount of support in the following passage from Kemble (p. 5):

"As a first step toward the formulation of such a description, we observe that a similar controversy arose long ago in connection with matter. In bulk it has properties which are conveniently described from the continuum point of view. In particular, it may be the vehicle of sound waves which act like waves in a continuous medium. If matter be molecular in structure, however, we must expect this fact to be most evident in the properties of



low density gases. Experiments made on such gases do favour the molecular hypothesis and are regarded as crucial since low density matter must in any case act in some ways like a continuum. Similarly, the corpuscular properties of radiation, if they exist, must be most evident if the corpuscles are of great energy and few in number as in the case of low intensity beams of hard X-rays. Precisely here the evidence for atomicity through the C. T. R. Wilson ray-track experiment and the Duane-Geiger point counter is most positive and definite. On the other hand, to get a test of one of the predictions of the wave theory one must have a record of the absorption of a quantity of light containing on the basis of the particle theory a very large number of photons."

The only difference between the above view and our own is that the particles and the waves are no longer the absolutely distinct and dissimilar entities, entirely unrelated to one another but are to be regarded as being essentially the same, one being only a constituent of the other. The establishment of this blood relationship between the two parties hitherto considered absolute strangers is regarded as a most satisfactory feature of the present theory. This relationship, it is suggested, may find another application in the theory of sound, in which the idea of "sound particles" (phonons) has been introduced and generally recognised, though there are some lurking doubts about their ultimate reality.

7. Here is, of course, a divergence from the accepted view that the energy of every beam of light of frequency  $\nu$  is  $h\nu$ . It is a challenge to the Einstein relation which has been shown in Chapter I to be an over-generalisation.



Our contention is that the law applies only to the complete photons but not to the partial photons. If this could be verified experimentally it would have been a crucial test. But the Quantum Mechanics postulates that it is impossible to make the particle and wave properties the subject of the same experiment. The energy and the frequency cannot be simultaneously determined. In our view, this postulate is easily understandable, because the two entities are distinct.

8. It may be pointed out that the equation of the partial photon offers a simple explanation of the finite breadth of the spectral lines. The expression

$$A (\sin k\xi - \sin k_1\xi)/\xi = A \int_{k_1}^k \cos k\xi dk$$

embraces a continuous range of simple harmonic waves and is equivalent to  $A\delta k \cos k\xi$  only approximately when  $k - k_1 = \delta k$  is small and  $\xi$  not too large. When  $\xi$  is large, the expression tends to zero.

It may be further observed that a train of simple harmonic waves has no natural beginning and no natural end. So if we conceive a particular atom radiating a photon in the form of a train of simple harmonic waves, we have to fix the number of oscillations artificially. For example, we may have to specify that the jump of an electron from one energy level to another in a hydrogen atom will emit a radiation of one million waves of a certain length. Otherwise an infinite train of waves, once started, will go on for ever. The expression  $(\sin k\xi - \sin k_1\xi)/\xi$  solves the difficulty as it tends to zero in the limit when  $\xi \rightarrow \infty$ .



Again, if in the train of simple harmonic waves  $A \delta k \cos k\xi$  possessing energy proportional to  $\delta k$ , the amplitude is halved (only by halving  $\delta k$ ,  $A$  being a universal constant), the phase remains sensibly unaltered, but the energy is also halved. **This gives a complete mathematical picture of the process followed in producing interference bands, where a beam of light is split up into two halves and made to recombine after one of them has been retarded.** It is a satisfactory feature of the theory that it fits in exactly with the technique of producing interference bands. It is usual to split up a beam with a half-silvered mirror. The two sections of the beam which fall on the two portions of the mirror respectively must, therefore, be regarded as parts of the same beam in the same phase and not two distinct rays. And this division is independent of the phase of the beam at the surface of separation. So, as photon succeeds photon each behaves in the desired way without any difficulty about the phase.

Another big point in favour of the theory is that it does not violate the principle of energy when a beam divides in the same medium unlike the elastic solid theory and the Maxwell electromagnetic theory.

---



## CHAPTER IV

### A NEW THEORY OF MATTER

1. Consider a train of electromagnetic waves of the type introduced in the last chapter  $E = A \sin k\xi/\xi$  and suppose it to be placed on a sphere in coils in the form of circles covering the entire surface. The exact manner of coiling is not of importance, but for fixing our ideas, we conceive the coil in the form of a loxodrome having an infinitely small angle of elevation from the equator.

[A loxodrome on the sphere is a curve which cuts the meridian planes at a constant angle which in the present case is very nearly a right angle. The coils may, therefore, be taken as circles coinciding with the parallels of latitude.]

Taking  $r, \theta, \phi$  for the usual spherical polar coordinates, the coils being parallels of latitude, let us place the electric vector  $E$  in the direction of  $r$  and the magnetic vector  $H$  in the direction of  $\theta$ .  $E$  has, therefore, the components,  $(E_r, 0, 0)$  at any point on the sphere and  $H$  the components  $(0, H_\theta, 0)$ .

We have to convert the Maxwell equations to the spherical polar coordinates. The components of curl in these coordinates, of a vector  $u$ , components  $u_r, u_\theta, u_\phi$  are

$$\frac{1}{r^2 \sin \theta} \left\{ \frac{\partial}{\partial \theta} (ru_\phi \sin \theta) - \frac{\partial}{\partial \phi} (ru_\theta) \right\},$$

$$\frac{1}{r \sin \theta} \left\{ \frac{\partial u_r}{\partial \phi} - \frac{\partial}{\partial r} (ru_\phi \sin \theta) \right\},$$



$$\frac{1}{r} \left\{ \frac{\partial}{\partial r} (ru_\theta) - \frac{\partial u_r}{\partial \theta} \right\}, \quad \dots \quad (1)$$

(Love, p. 56.)

The six Maxwell curl equations reduce to four.

$$\frac{1}{\varpi} \frac{\partial E_r}{\partial \phi} = -\frac{1}{c} \frac{\partial H_\theta}{\partial t}; \quad -\frac{1}{\varpi} \frac{\partial H_\theta}{\partial \phi} = \frac{1}{c} \frac{\partial E_r}{\partial t} \quad \dots \quad (2)$$

$$-\frac{1}{r} \frac{\partial E_r}{\partial \theta} = 0; \quad \frac{1}{r} \frac{\partial (rH_\theta)}{\partial r} = 0 \quad \dots \quad (3)$$

$\varpi$  being the perpendicular on the axis  $= r \sin \theta$ .

It is easily verified that  $E_r = H_\theta = f(\varpi\phi - ct)$  satisfy the equations along the coils (equations 2). Since the coils are distinct there is no continuity in the direction of  $r$  and  $\theta$  and the differential coefficients in these directions do not exist.

The two divergence equations are, however, not satisfied. This is not entirely unexpected. For in the model of an electron, the effect of the charge must now come in.

2. It is proposed that all fundamental particles which have a charge  $\pm e$  are represented by photons

$$E = A \sin k(s - ct)/(s - ct),$$

where  $s$  denotes the length of the arc measured from the hump of the photon. Then the total energy is  $\frac{1}{2}A^2\mu^2k = h\nu$ , assuming that there has been no change (or only a small change) of energy when a linear photon is coiled. This identifies the mass of the particle with energy



and establishes connection with de Broglie's theory which lays down that a particle of mass  $m$  possesses the character of a wave group of frequency  $\nu$  where  $mc^2 = h\nu$ . A particle, therefore, differs from a photon only in the form of the electromagnetic wave. The former is a compact formation while the latter is linear, extending to infinity on both sides.

The following quotation from Jeans (I, p. 54) gives an interesting picture of the theory of Faraday and Maxwell which has a bearing on the present context. "They pictured an electrified particle as an octopus-like structure, a small concrete body which threw out a sort of feelers or tentacles, called lines of force, throughout the whole of space. When two electrified particles attracted or repelled one another, it was because these tentacles had somehow or other taken hold of one another, and pushed or pulled. These tentacles were supposed to be formed out of electric and magnetic forces, of which radiation is also formed. When an electron emitted radiation it merely discharged some of its tentacles into space, much as a porcupine is said to throw out its quills." Modern Physics regards the shedded quills as part of the concrete body, for matter and energy are the same and the deviation of light by matter is a proved fact.

3. The present theory solves at once the problem of solar energy. Annihilation of matter and transformation into energy cease to be mystic phenomena and are to be regarded as only a change in form. In the intense heat in the interior of the sun, protons and electrons are being transformed into radiation. It may be surmised that the



short wave radiations are as the usual rule reflected back while the longer waves are transmitted in the form of light and heat. The shorter waves which are deadly to life find another barrier at the outer atmosphere of the earth, which makes life possible. Some of the shorter radiations, it is suggested, filter through and form the cosmic rays. These in crossing two barriers have lost their sense of direction and are merely floating away at random in space. This is one of the unexplained peculiar features of these rays.

It may also be tentatively suggested that the variations in the brilliance of the cepheid variables may be due to some unstable conditions owing to the imprisoned short wave radiations. These would exert a pressure on the outer layers in trying to escape, while they are held back by gravitation and turned back by reflection. It is probable that the variables are suns which have not reached the steady state. By the way, the current view of the period of the cepheid variables being related to their distances from the earth seems to a layman to be reminiscent of the Ptolemaic idea of the earth being the centre of the universe and man the lord of creation!

This model also explains Thomson's experiment on the diffraction of electron waves. It does not seem to be definitely known whether these have finite impulse and energy. This problem does not arise if the Einstein relation be accepted, for then every electromagnetic wave of frequency  $\nu$  possesses finite energy and impulse. But it does on the present theory. If the energy be finite, the diffraction pattern is to be supposed to be caused by complete linear photons of energy  $h\nu$ . But if the energy

be infinitesimal, the electron must be supposed to have shed a simple harmonic wave  $A \delta k \cos k(x - ct)$  which gives the diffraction effect.

4. One of the early triumphs of the Quantum Theory was Bohr's theory of the hydrogen spectrum. It can be shown that the concept of the quantum is not essential.

Bohr envisages a number of energy levels, for the electron in a hydrogen atom for instance and the passage from a higher to a lower level is attended with the emission of energy  $W_m - W_n = h\nu_{mn}$ , where  $\nu_{mn}$  is the frequency of the emitted wave.

Taking the simplest model of a hydrogen atom, the electron moving in a circular orbit, we have the following formulae:—

$$ma^2\omega = nh/2\pi, \quad \dots (1)$$

$$a = \frac{n^2 h^2}{4\pi^2 m e^2}, \quad \omega = \frac{8\pi^2 m e^4}{n^3 h^3}. \quad \dots (2)$$

The periodic time  $\tau = \frac{2\pi}{\omega} = \frac{n^3 h^3}{4\pi^2 m e^4}, \quad \dots (3)$

and the frequency of the electron in its orbit is  $1/\tau$ .

(Sommerfeld, p. 706.)

$$W = -\frac{2\pi^2 m e^4}{h^2 n^2} \quad \text{gives for different integral values of } n$$

the energy levels. Substitution in the Bohr formula leads at once to the Balmer formula

$$N = \frac{v}{c} = R \left( \frac{1}{n^2} - \frac{1}{m^2} \right),$$



where  $N$  is the wave number and  $R = \frac{2\pi^2 m e^4}{ch^3}$ .

We retain the energy levels with Bohr's fundamental assumption that no energy is radiated when the electron is revolving at that level contrary to the conclusion of the old classical theory that energy is always radiated when the motion is accelerated. In the matter of passage from one level to another we regard initially  $n$  in the above formulae (1—3) as a number, not as yet an integral quantum number. We introduce the concept of an effective frequency

$${}^{''}\nu^{''} = \int_n^m \frac{1}{\tau} du, \text{ where } m \text{ and } n \text{ in the limits are integral}$$

quantum numbers. So we have

$${}^{''}\nu^{''} = \int_n^m \frac{2\pi}{h} E du = \frac{4\pi^2 m e^4}{h^3} \int_n^m \frac{dn}{n^2} = cR \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$$

giving the wave number  $N = R \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$ .

This is Balmer's formula.

The idea of effective frequency is new and is justified on the ground that it removes the logical contradiction of the more straightforward Bohr formula. It becomes the *average* frequency when the quantum numbers  $m, n$  differ by 1. This may have some connection with the selection rules. It revives the now forgotten principle of the old classical theory that the frequency of the generating electron is the same as the frequency of the light emitted. It also supplies the connection of the red shift of the



Relativity theory with the frequency of the electron. **On Bohr's theory Einstein's derivation of the red shift has no meaning.** The energy of the waves emitted belong to the class of partial photons of energy  $h\delta\nu$  and the fall of the energy from one level to another, is made up by the number of photons emitted.

If this thesis be accepted, Bohr's formula becomes a handy rule of thumb operation for determining the frequency without its illogical consequences.

5. To find the charge on an electron, we have to apply Gauss theorem that the total normal induction is  $4\pi$  times the charge enclosed

$$4\pi e = \int N dS.$$

The normal induction  $N$  is equal to the electric intensity regarded as surface density.

Replace the coils of our model by thin strips of breadth  $\delta$ , so as to cover the entire spherical surface. We spread out the electric force  $E_r$  over the width of the strip. The normal induction is thus  $E_r/\delta$  and the elementary area  $\delta ds$ . We have, therefore,

$$4\pi e = \int N dS = \int \frac{E_r}{\delta} \delta ds = A \int_{-\infty}^{\infty} \frac{\sin k(s-ct)}{s-ct} ds = \pi A.$$

This gives the value  $\frac{1}{4} A = 2.8 \times 10^{-9} \mu^{-1}$  for the electronic charge against the experimental figure  $4.8 \times 10^{-10}$  c.s.u. We can thus get a definite value for  $\mu$ . **But the main interest of the result lies in the fact that the figure is a constant and independent of  $k$ , which determines the mass of**



**the particle.** An electron and a positron must have the same charge, positive or negative. And so must have the proton and the various mesons.

If again, we have a partial photon of the form  $A(\sin k\xi - \sin k'\xi)/\xi$  the charge would be zero. Neutrons, neutrinos and the recently discovered antineutrons all come under this category.

Moreover, Gauss' theorem is a direct consequence of the inverse square law. It is obvious that this model of an electrically charged particle would behave as if it was exerting a force varying as the inverse square of the distance. The charge is thus a derived property of matter and is not conserved in the same manner as mass or energy. If a photon coil becomes uncoiled, the charge automatically disappears. Why photons of certain wave lengths have the tendency to form coils we have no idea. But it can be shown from general considerations that if a charged particle be stable, the corresponding particle of opposite sign must be unstable. If  $V$  be the potential energy of the former,  $-V$  will be the potential energy of the latter. For stability, the first differential coefficient of  $V$  with respect to the independent variable, whatever it may be, must be zero and the second derivative negative. This would make the second derivative of the potential energy of the second particle positive, making it unstable.

6. It would be of interest to recall here Dirac's theory of the positron (Dirac, p. 270). It is intimately connected with the idea of negative energy states which arise from the fact that "in the Relativity theory, the energy of a free particle is given by a square root

$$E = \pm \sqrt{(p^2 + m^2 c^4)},$$



where  $p$  is the momentum. Thus for any value of  $p$ , the sign of  $E$  may be either positive or negative.

“ The existence and properties of these negative energy states give rise to serious difficulties. The acceleration of such an electron due to an external force is in the opposite direction of the force. Thus an electron in negative energy states would repel an ordinary electron but would itself be accelerated towards the electron in the positive energy state. The principle of action and reaction would not be valid.

“ In the classical theory, however, no difficulty arises, because one can define energy to be the positive square root; and then it does not change with time. This is not possible in the Quantum Theory. . . . . The latter (the negative energy states) cannot be excluded.”

Says Dirac (p. 271): “ We cannot, however, simply assert that the negative energy solutions represent the positrons, as this would make the dynamical relations all wrong. For instance, it is certainly not true that a positron has negative kinetic energy. We must, therefore, establish the theory on a somewhat different footing. We assume that all the negative energy states are occupied, with one electron in each state in accordance with the exclusion principle of Pauli. An unoccupied negative energy state will now appear as something with positive energy. . . . . We assume that these unoccupied negative energy states are the positrons.

“ These assumptions require there to be a distribution of electrons of infinite density everywhere in the world. A perfect vacuum is a region where all the states of positive



energy are unoccupied and all those of negative energy occupied.

“ In a perfect vacuum Maxwell's equation

$$\operatorname{div} \mathcal{E} = 0$$

must of course be valid. This means that the infinite distribution of negative energy electrons does not contribute to the electric field. Only departures from the distribution in a vacuum will contribute to the electric density  $\rho$  in Maxwell's equation

$$\operatorname{div} \mathcal{E} = 4\pi\rho.”$$

7. The hole theory of the position is open to the objection that it upsets the entire electromagnetic field. Dirac's assumption of calculating the electric field by its excess over the field of electrons in all the negative energy states satisfies the linear Maxwell equations no doubt. But there are other relations which are not linear, *e.g.*, the energy distribution of the field. It is not possible to keep the initial field of electrons in all the negative energy states entirely out of the picture.

The difficulties of the theory have been further explained by Heitler as follows (p. 191):—

“ The definition of the negative energy states depends upon the electro-magnetic field. . . . . If we consider an electro-magnetic field, for instance, the Coulomb field of a hydrogen nucleus, the negative energy states are not the same as for a free electron. If the wave function of a stationary state of the hydrogen atom is expanded in a series of plane waves, the positive and negative energy levels become mixed. Therefore, if all negative energy



states of the free electron are filled up, some of the negative energy states of the hydrogen atom will be unoccupied, while some of the positive energy states will be occupied. If, for a moment, we imagine the Coulomb field switched on adiabatically, a certain number of electron pairs would thus be created, which means that the vacuum could be polarised by an electro-magnetic field. . . . .

“ Also the idea of an electro-magnetic field in vacuo has to be abandoned. Even if no particles are present, an electro-magnetic field can give rise to the creation of pairs. Since, however, for this purpose a minimum energy  $2mc^2$  is required, pairs can only be created if in the Fourier expansion of the field, frequencies higher than  $2mc^2/\hbar$ , or wave lengths smaller than  $\lambda_0/2$  occur. If this is the case, a pure field in vacuo no longer exists. . . . .

“ That a decisive change must occur in the laws of ordinary electro-dynamics is evident also from the following consideration: The creation of pairs in intermediate states gives rise to some processes which are impossible in principle according to ordinary electro-dynamics.” For further details reference may be made to the treatise.

8. We may also make a rough calculation of the size of the electron from the known values of the angular momentum  $\hbar/2\pi$  ( $\sim mca$ ).

We get radius  $a = 2 \times 10^{-11}$ . The value obtained on the classical theory of the mass being entirely due to moving electrical charges is  $2 \times 10^{-13}$ . The hypothesis has, however, long been discarded. It may be observed here that with a mechanical model, the classical figure for the radius of the electron would make the velocity at the periphery



greater than the velocity of light—a contingency unthinkable to a relativist. It will have to be remembered that the electrons are no longer of the billiard ball type, so a certain amount of penetration of one particle by another is not ruled out. But the main point is that the problem of infinite energy resolves itself. There is no longer a point singularity at the centre.

9. One of the basic conclusions of the Relativity theory is that the mass of a moving particle, velocity  $v$ , is given by  $m_0/(1 - v^2/c^2)^{1/2}$  where  $m_0$  is the rest-mass.

No theory of the structure of an electron has had anything to say on this problem. The linear wave packet theory of Schrödinger gives, as we have seen in Chapter III, an entirely wrong expression. To explain the apparent change of mass due to motion relative to an observer, we recall the formula for the transverse Doppler effect. Consider a photon given by the expression

$$A \sin k(y' - ct')/(y' - ct'),$$

the origin  $O'$  of the dashed co-ordinates having a velocity  $v$  in the direction of the  $x$ -axis. The Lorentz transformation is

$$x' = \beta(x - vt), \quad y' = y, \quad z' = z, \quad t' = \beta(t - vx/c^2).$$

Then the trigonometrical factor which gives the new frequency becomes

$$\sin k\{y - c\beta(t - vx/c^2)\} = \sin k(y + \beta vx/c - c\beta t).$$

The apparent frequency to the observer at  $O$  is  $\nu/(1 - v^2/c^2)^{1/2}$ ,  $\nu = hc/2\pi$  being the frequency to the observer at  $O'$ . The longitudinal Doppler effect has already been considered in Chapter III.

Consider now an electron moving with velocity  $v$  relative to the observer at  $O$  with its axis perpendicular to the direction of  $v$ .  $m_0$  is the mass and  $\nu$  the frequency of the electromagnetic wave to an observer stationed on the electron so that  $m_0 c^2 = h\nu$ . Take a coil with its plane parallel to the direction of  $v$ . At the extremities of the transverse diameter, the Doppler effect is longitudinal and the apparent frequencies

$$\nu \sqrt{(1 - v/c)/(1 + v/c)} \text{ and } \nu \sqrt{(1 + v/c)/(1 - v/c)}.$$

The mean frequency is, therefore,

$$\nu/(1 - v^2/c^2)^{\frac{1}{2}}.$$

At the ends of the longitudinal diameter, the Doppler effect is transverse and the apparent frequency is again  $\nu/(1 - v^2/c^2)^{\frac{1}{2}}$ . At an intermediate point  $A$  on the coil, both the effects are simultaneously present and are complementary. If  $\alpha$  be the angle which the radius to  $A$  makes with the direction of  $v$ ,  $v$  can be replaced by its two components,  $v \cos \alpha$  along the tangent and  $v \sin \alpha$  along the radius. The former produces the Doppler effect  $\nu \sqrt{(1 - v \cos \alpha/c)/(1 + v \cos \alpha/c)}$  on the frequency. If we consider simultaneously the diametrically opposite point  $B$  on the coil, the corresponding Doppler effect there is  $\nu \sqrt{(1 + v \cos \alpha/c)/(1 - v \cos \alpha/c)}$ .

The mean of these two is

$$\nu/(1 - v^2 \cos^2 \alpha/c^2)^{\frac{1}{2}}.$$

The radial component of velocity gives the Doppler effect as

$$\nu/(1 - v^2 \sin^2 \alpha/c^2)^{\frac{1}{2}}.$$



The total effect is an apparent change of frequency from  $\nu$  to  $\nu/(1-v^2/c^2)^{\frac{1}{2}}$  correct to the square of  $v/c$  and the mass from  $m$  to  $m/(1-v^2/c^2)^{\frac{1}{2}}$  to the same order of approximation.

Consider in the second place an electron moving in the direction of the axis. The plane of the coils is perpendicular to  $v$ , and at every point of the coil, the Doppler effect is transverse. The frequency, therefore, changes from  $\nu$  to  $\nu/(1-v^2/c^2)^{\frac{1}{2}}$ , and the mass from  $m$  to  $m/(1-v^2/c^2)^{\frac{1}{2}}$ . Combining these two cases we get the usual formula for the apparent change of mass due to motion relative to any observer.

10. The last property of matter we shall deal with is gravitation. There is no theory extant which seeks to explain it. What is suggested here must be taken very tentatively.

It is well known that gravitation as a force is very much weaker than the Coulomb force. Between two electrons, the latter is  $(4.7 \times 10^{-10})^2 = 2.2 \times 10^{-19}$  at a distance of 1 cm. The force of gravitation if there is one between them is  $6 \times 10^{-8} \times 81 \times 10^{-54} = 4.8 \times 10^{-61}$  at the same distance. It will be recalled that in a picture of the structure of matter, protons of mass 1840 times that of electrons form the major constituent of the nucleus where the greater part of the mass is concentrated. About the same number of electrons are arranged in shells at different distances from the nucleus.

These shells of electrons form a sort of cloud screen, but not quite unbroken. Through these breaks come out the lines of force, very few in number it is true, but still



they do. The effect of these come out as pulls at and above a certain distance, but at shorter distances the repulsion of the electrons of the outer shells of two neighbouring atoms is predominant. Gravitation may thus be regarded as the *bulk* effect of these straggling lines of force. A certain amount of penetration of one atom by another at the outer periphery is not ruled out.

---



## CHAPTER V

### SUMMARY AND CONCLUSIONS

1. Starting from the Maxwell equations in free space, it has been proved that a linear wave along the  $x$ -axis which presents the Doppler effect and is consistent with the Relativity Theory must have an equation of the form

$$E_y = H_z = A \sin k(x - ct)/(x - ct)$$

where  $A$  is a universal constant, the other components of the electric and magnetic vectors being zero. To ensure the unique nature of the path of the beam these forces are restricted to act at points on the path of the beam itself and the wave packet is identified with a photon. This represents a train of waves of length  $2\pi/k$  of diminishing amplitudes extending in either direction to infinity (where the amplitude vanishes) with a hump in the middle. It may also be regarded as a bundle of simple harmonic waves of all frequencies ranging from 0 to  $kc/2\pi$ . The energy is given by a line integral and is found to equal  $h\nu$  and the impulse  $h\nu/c$  on the assumption of the existence of a limit. It is emphasised that the Einstein relation  $W = h\nu$  must be regarded as an over-generalisation.

It is then verified that a photon can be split up into a photon of less energy and a partial photon of the form

$$A \{\sin k(x - ct) - \sin k_1(x - ct)\}/(x - ct)$$

of energy  $h(\nu - \nu_1)$ . The latter reduces to a simple harmonic wave  $A\delta k \cos k(x - ct)$  where  $k - k_1 = \delta k$  is small, and



$x - ct$  not too large. The energy is also proportional to  $\delta k$ . This explains the breadth of the spectral lines and the ultimate natural fading out of light. Further if the amplitude of the harmonic wave be halved the energy is also halved. This corresponds exactly to the method followed in producing interference bands. The current controversy between the wave and particle nature of light is, therefore, settled. An electron is conceived as a photon in the form of coils along the parallels of latitude on a sphere. This agrees with the de Broglie theory and gives in particular the electron waves. By applying the Gauss equation, the charge is found to be a constant. The inverse square law follows as a consequence and the problem of infinite energy resolves itself. This charge is independent of the mass of the particle. Lorentz transformation leads to the formula for the apparent change of mass with velocity.

2. It must be emphasised that the method is deductive and unless the premises are disputed, the conclusion cannot be denied. Since the Maxwell equations and the Relativity theory are universally acknowledged foundations of modern Physics, the form of the equations of the photons must be admitted; unless of course physicists are prepared to discard Aristotle's logic just as Galileo disproved Aristotle's physics. The question then becomes one of fitting this in the scheme of experimental results. A physical picture of a photon is given which leads on certain assumptions to Planck's Law but the picture is not of vital importance. What is essential is that the energy of the beam should be represented by a line integral along



the length of the beam. The expression for the impulse easily follows.

It may be worth while here to note Jeans' warning that "the attempt to regard the flow of energy as a concrete stream always defeats itself. . . . The concept of energy flowing about through space is useful as a picture, but leads to absurdities and contradictions if we treat it as a reality. Professor Poynting gave a well-known formula which tells us how energy may be pictured as flowing in a certain way, but the picture is far too artificial to be treated as a reality. . . . The mathematician brings the whole problem back to reality by treating the flow of energy as a mere mathematical abstraction" (I, pp. 108-109).

The divisibility of the beam is then proved without doing violence to the energy principle. This brings us to the parting of ways with modern Physics based on the Einstein relation. Various arguments have been adduced to prove that it is an over-generalisation. The mathematical process follows step by step the experimental process of producing interference bands.

While the above theory of the photon is based on the firm foundation of Maxwell equations and the Relativity theory, the theory of electrons is, however, more tentative and we freely admit, rather naive. Its justification lies in the well-known facts of experiment which it explains. The main feature of the theory is the complete identification of matter and energy and the bridge it throws over the gulf separating waves and particles. Its non-compliance with two of the Maxwell equations has already been referred to. This is perhaps not altogether unexpected, for



here we have been brought face to face with the charge. Only the density of charge has no meaning in our theory of discrete charges. It may also be pointed out that the two divergence equations of Maxwell are rather difficult to interpret when we leave the old classical standpoint of continuous distribution. The theory of charge is only a re-iteration of the Maxwell equation  $\text{div } E = \rho$ . Only this equation is very often lost sight of and the charge treated as if it was a part of the fundamental concept of matter instead of being only a derived property. Moreover, all the fundamental particles, electrons, positrons, neutrons and protons are placed on the same footing, their different masses being due to the different range of frequencies of waves in their composition. The different signs of the charges are explained by the opposite direction of the electric vector. Mesons also fall in line with the scheme, possessing varying masses but possessing, as far as can be ascertained, the same charge. If the theory is correct, pair production is not due to the conservation of the charge, but to the conservation of angular momentum.

3. Another interesting feature of the new theory is that the radiation from an electron when there is a transfer from one energy level to another, comes from the corpus of the electron itself. It will be recalled that the Bohr theory of the spectrum which runs right through the Quantum Mechanical theory of radiation is based on the Newtonian conception of potential energy. But the Relativity theory has introduced a new conception of field of force. According to the latter, in free space, the distance between two neighbouring points in the four dimensional



continuum of space and time is given by the equation

$$\begin{aligned} ds^2 &= c^2 dt^2 - dx^2 - dy^2 - dz^2 \\ &= c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \end{aligned}$$

the signs on the right hand side being a matter of convention. It is quite permissible to take the time with the negative sign and the space coordinates with the positive. But if a particle of mass  $m$  is introduced at the origin, the metric is altered to

$$ds^2 = -\left(1 + \frac{2m}{r}\right) dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + c^2 \left(1 - \frac{2m}{r}\right) dt^2.$$

The space may be supposed to have been distorted by the introduction of the particle and the gravitational force is only the apparent consequence of this distortion.

That the Relativity theory mixes up the potential and the kinetic energies can easily be seen from the following example: consider the mass of a particle at rest to an observer  $O$ . If this observer has a velocity  $v$  with reference to another observer  $O'$ , the mass of the particle to the latter in energy units is  $m_0 c^2 / (1 - v^2/c^2)^{1/2} = m_0 c^2 + \frac{1}{2} m_0 v^2$ . The second term represents the kinetic energy of classical mechanics which has been introduced by a change of position of the observer himself.

In fact, the idea of a field of force as a vector field is, strictly speaking, foreign to the basic idea of the Relativity theory, but has to be used for progress and must be regarded as an approximation. The Relativity mathematics, except for the simplest problems, is extremely complicated. As Eddington has remarked, the problem of two bodies remains yet a challenge to the relativistic mathematician in much



the same way as the problem of three bodies to the classical mathematician.

**The idea of radiations emanating from the corpus of the electron is, broadly speaking, more in conformity with the Relativity theory than Bohr's idea based on the classical Newtonian potential.** It presents again a more satisfactory means of conveyance of excess energy than Fermi's neutrinos.

A tentative explanation may be offered here for the non-absorption of radio waves by matter. The power of absorption of radiation by electrons, on the present theory, depends on the "hunger" of these electrons for the special radiations. If a large number of electrons are lacking in certain waves in their composition, they will readily absorb radiations of that wave length. Thus visible light is readily absorbed by all matter, while metals with a large number of free electrons absorb radiations more freely than non-conductors. Radio waves can penetrate thick walls because these radiations are usually not missing from the composition of the electrons and are thus transmitted without change. **This property, therefore, depends not so much on the amount of energy of the radiations but on the "hunger" of the electrons for those radiations.**

Lastly, Lorentz transformation gives the longitudinal and transverse Doppler effects. This gives, on the basis of our model of the electron, the Relativity variation of mass which has hitherto been regarded as a postulate. There is no theory extant which even attempts at an explanation.

4. Why photons of certain wave lengths should have the tendency to form coils, some stable others unstable,



we have no idea. The reason can only be understood when we have better insight into the interaction between the different parts of the photon and a more complete picture of their structures. There must be a number of knotty points which will have to be settled before the theory can be regarded as fully established. But if it can stand these tests, modern Physics will have been brought down from the clouds to the base but solid earth. If simplicity be one of the tests of truth, we hope it will be agreed that our model of the fundamental particles has greater claim to recognition than the nebulous and intangible abstractions of modern Physics.

### REFERENCES

- Dirac, P. A. M., *The Principles of Quantum Mechanics*, Oxford, 1935.
- Frenkel, W., *Wave Mechanics, Elementary Theory*, Oxford, 1932.
- Heitler, W., *The Quantum Theory of Radiation*, Oxford, 1936.
- Jeans, J. H., I. *The Mysterious Universe*, Cambridge, 1930.  
 II. *The Mathematical Theory of Electricity and Magnetism*, Cambridge, 1911.
- Kemble, E. C., *Fundamental Principles of Quantum Mechanics*, New York, 1937.
- Lamb, H., *Hydrodynamics*, Cambridge, 1906.
- Love, A. E. H., *Theory of Elasticity*, Cambridge, 1906.
- Richtmyer, F. K., & Kennard, E. H., *Introduction to Modern Physics*, New York, 1947.
- Sommerfeld, A., *Atombau und Spektrallinean*, Braunschweig, 1924.
- Tolman, R. C., *Relativity Thermodynamics and Cosmology*, Oxford 1934.
-